

Short Course of Electrochemistry

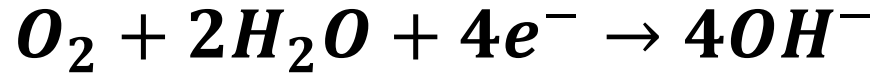
L3: Electrochemical Impedance Spectroscopy

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Electrochemical Reaction



- **Faradaic** current density

$$i_F = n_{O_2} F k_{O_2} c_{O_2} \exp \left[\frac{\alpha_{O_2} F}{RT} (E - E^0) \right]$$

- **Non-faradic** current density

$$i_{non-F} = C_d \frac{dE}{dt}$$

- Total current density = Faradaic + Non-faradaic

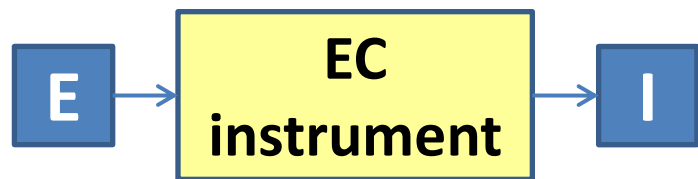
$$i_{total} = i_F + i_{non-F}$$

- Cell potential = Electrode potential + iR drop

$$U = E + iR_s$$

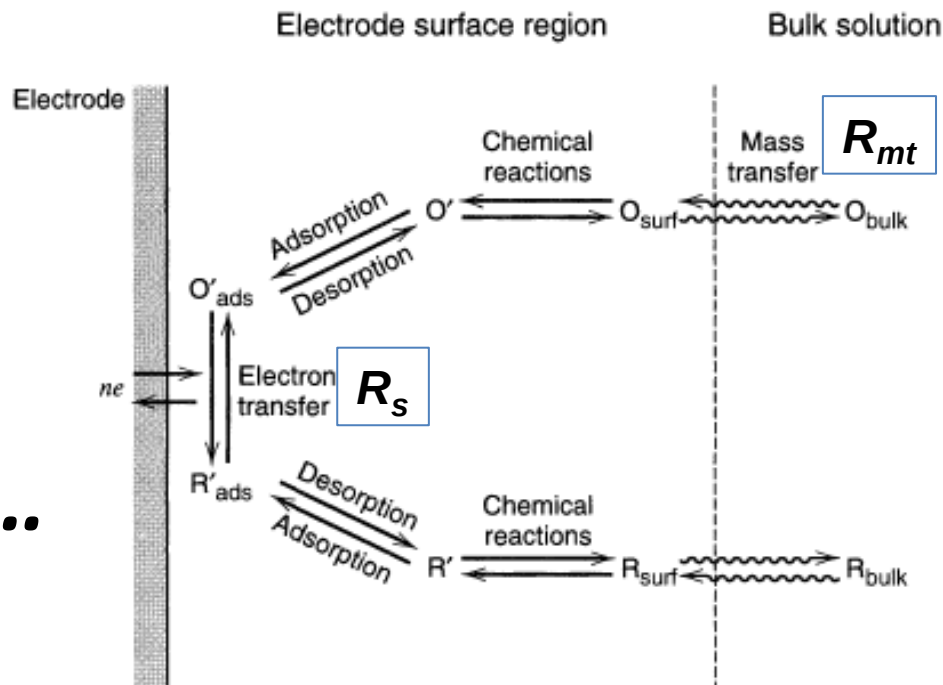
DC Technique

Direct current: $E = I \times R$

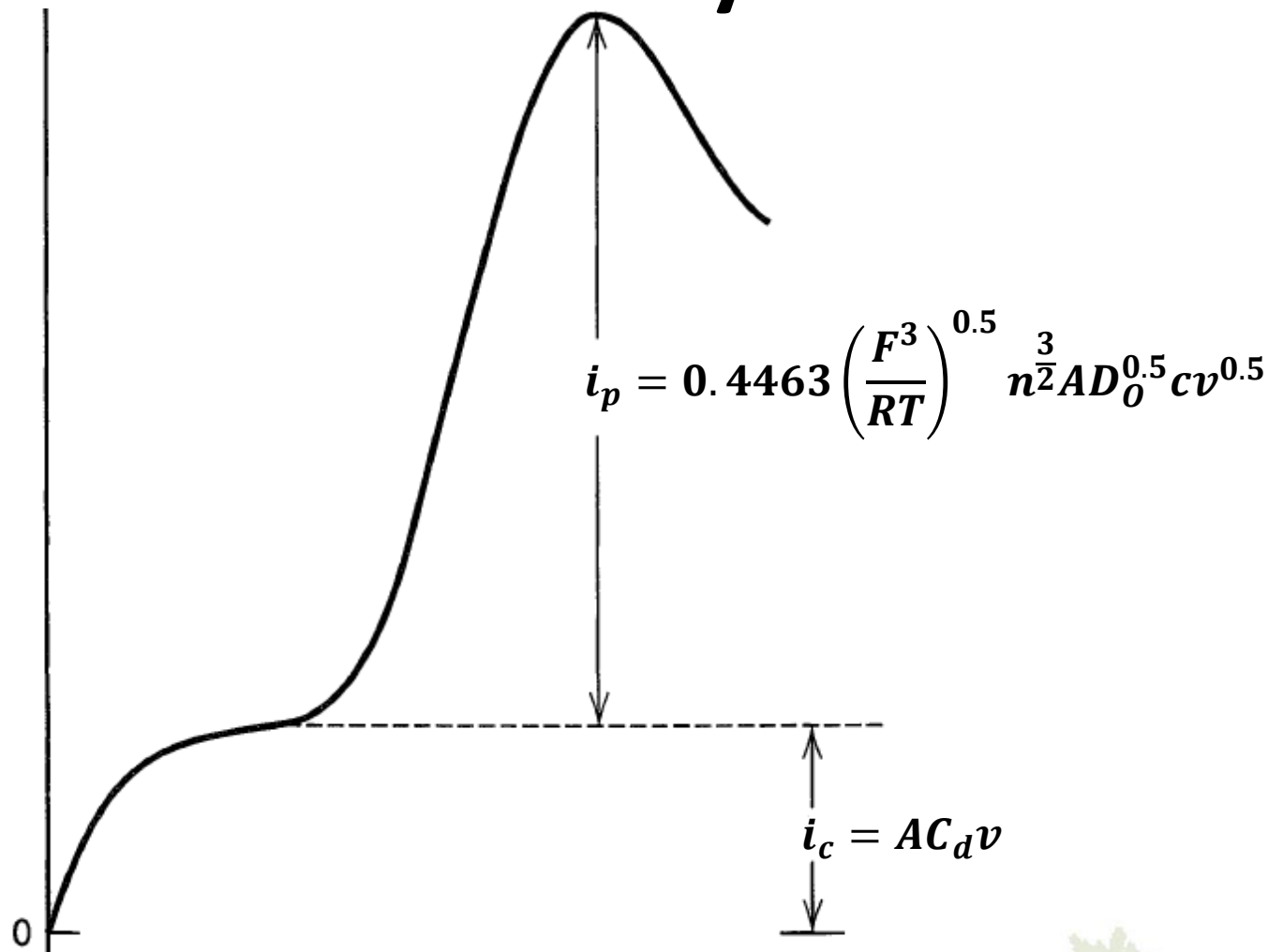


$$R = R_{mt} + R_{ct} + R_s + \dots$$

$$I = I_F + I_{non-F}$$



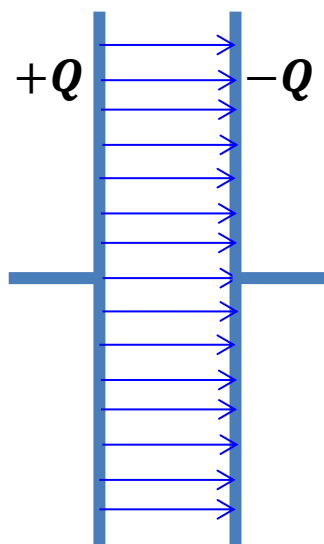
DC Method: Linear Sweep Voltammetry



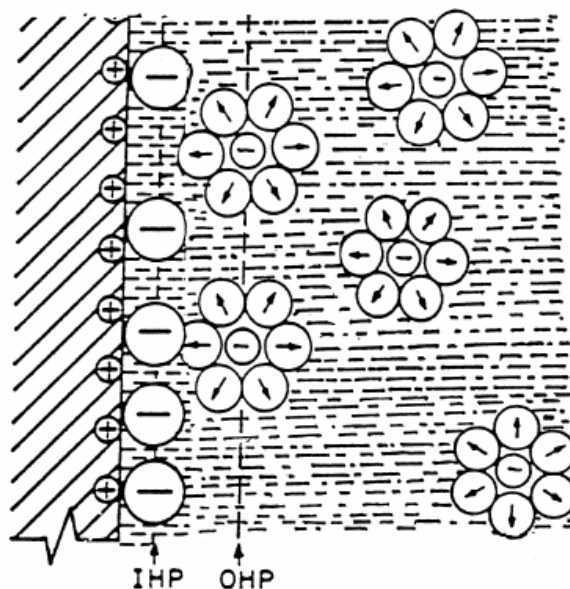
How to get each parameter in a simply way?

Electrochemical Features and Electrical Elements

- Resistor, R : Electron-transfer resistance, solution resistance...
- Capacitor, $C \rightarrow$ Double layer capacitance



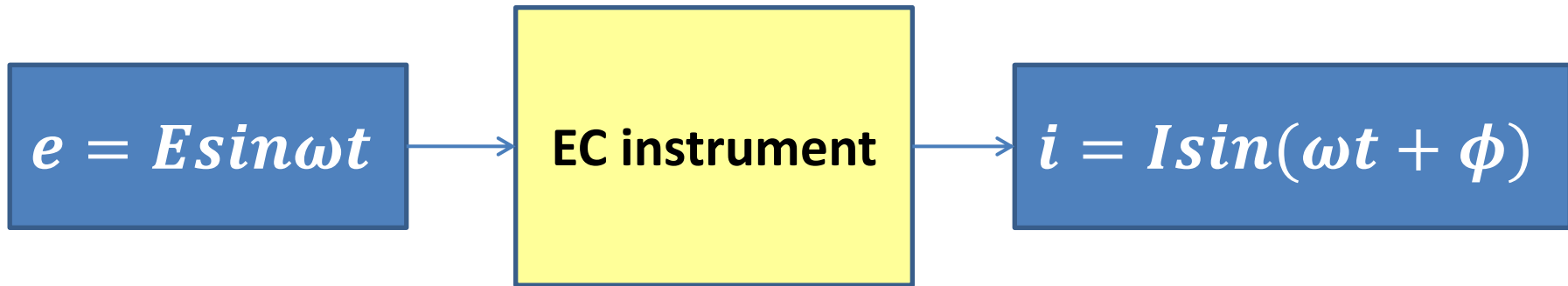
Electric field: ϵ



Double layer between electrode and solution.
IHP: inner Helmholtz plane
OHP: outer Helmholtz plane

AC Technique

Alternating current: $e = i \times \mathbf{z}$



Impedance: $\mathbf{z} = \frac{e}{i} = \frac{E \sin \omega t}{I \sin(\omega t + \phi)}$

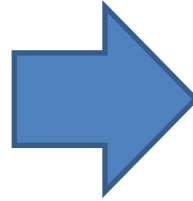
Input and Response

Input: Potential wave

$$e = E \sin \omega t$$

$$\omega = 2\pi f$$

f : Frequency



Response: Current wave

$$i = I \sin(\omega t + \phi)$$

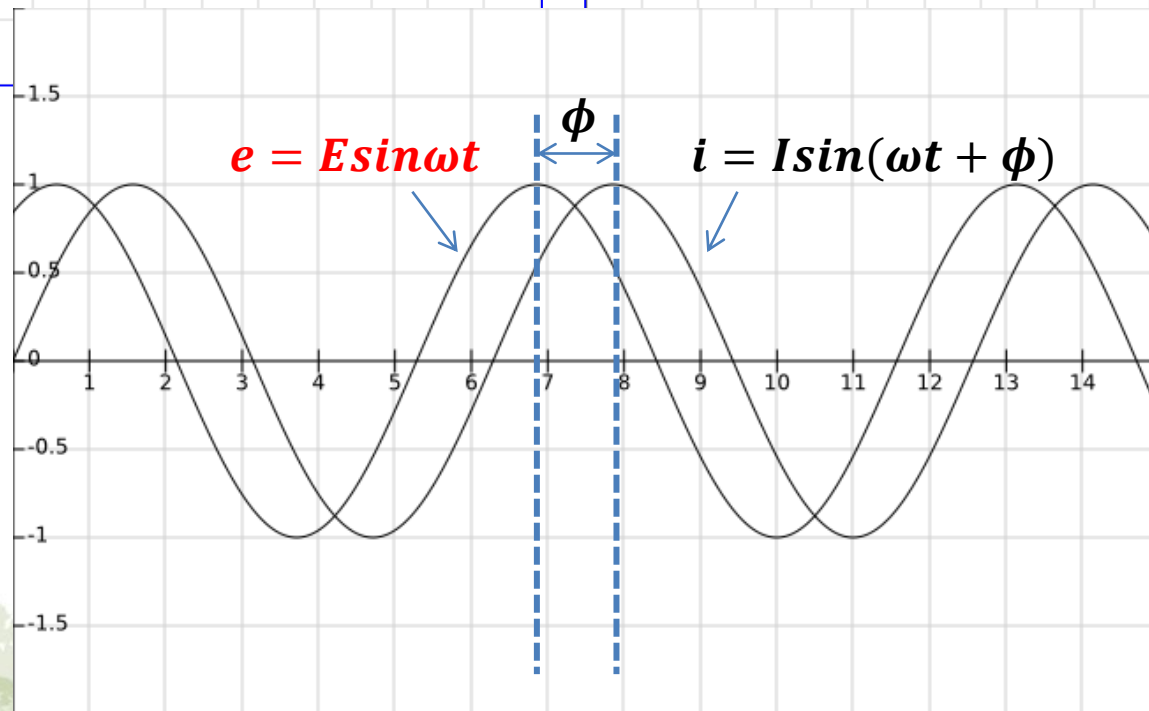
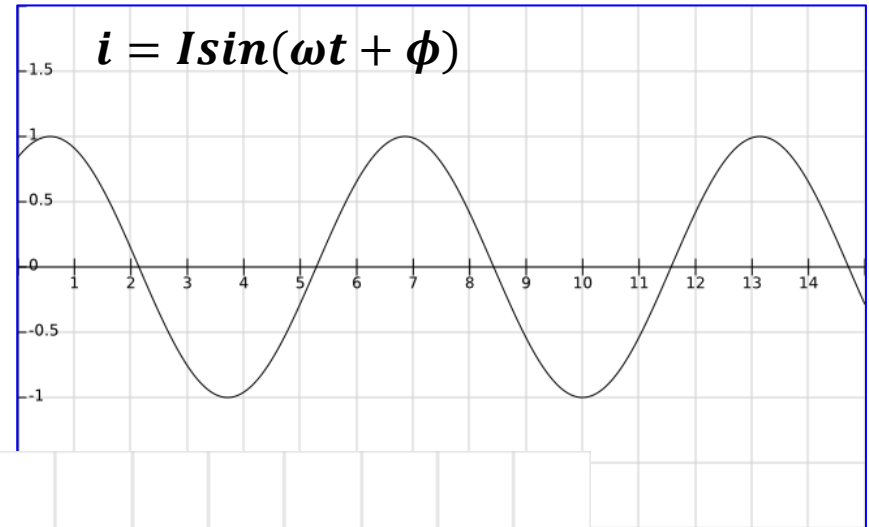
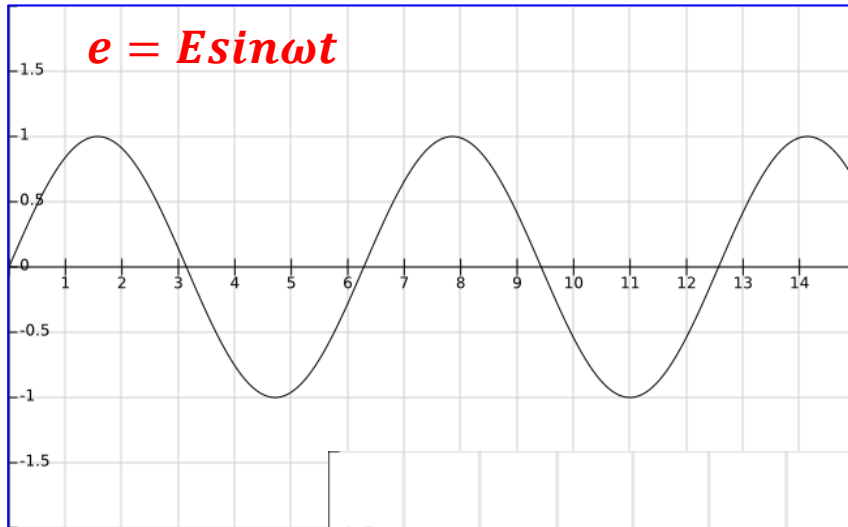
ϕ : Phase shift



Impedance wave

$$z = \frac{e}{i} = \frac{E \sin \omega t}{I \sin(\omega t + \phi)}$$

Phase Shift



Euler's formula: Complex form

$$\left. \begin{aligned} e &= E \sin \omega t \\ i &= I \sin(\omega t + \phi) \\ z &= \frac{e}{i} = \frac{E \sin \omega t}{I \sin(\omega t + \phi)} \end{aligned} \right\} f(\sin x, \cos x)$$

Real part	Imaginary part
$e^{jx} = \cos x + j \sin x$	
$j = \sqrt{-1}$	

Complex From

- Input:

$$e = E(\cos\omega t + j\sin\omega t) = Ee^{j\omega t}$$

- Response:

$$i = I[\cos(\omega t + \phi) + j\sin(\omega t + \phi)] = Ie^{j(\omega t + \phi)}$$

- Impedance:

$$Z = \frac{e}{i} = \frac{Ee^{j\omega t}}{Ie^{j(\omega t + \phi)}} = \frac{E}{I}e^{-j\phi} = R \times e^{-j\phi} = R(\cos\phi - j\sin\phi)$$

$$Z = \mathbf{Z'} + j\mathbf{Z''} = \mathbf{Z_{Re}} - j\mathbf{Z_{Im}} = Re^{-j\phi}$$

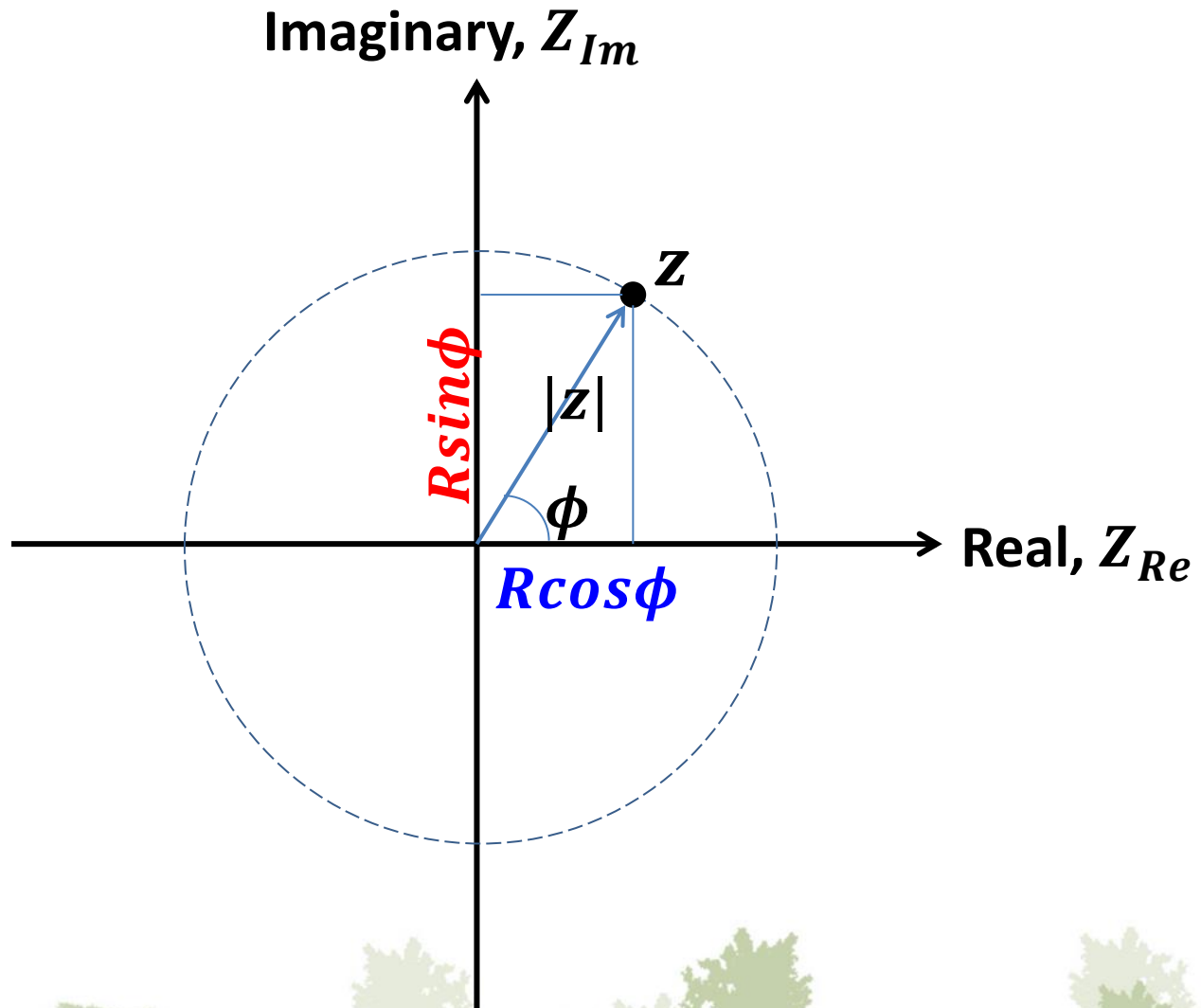
- Absolute value or length of impedance

$$|Z| = \sqrt{|Z_{Re}|^2 + |Z_{Im}|^2}$$

- Phase angle

$$\tan\phi = \frac{Z_{Im}}{Z_{Re}} \Rightarrow \phi = \tan^{-1} \frac{Z_{Im}}{Z_{Re}}$$

Complex Plane



Resistor

- Input:

$$e = E \sin \omega t$$

- Response:

$$i = I \sin \omega t$$

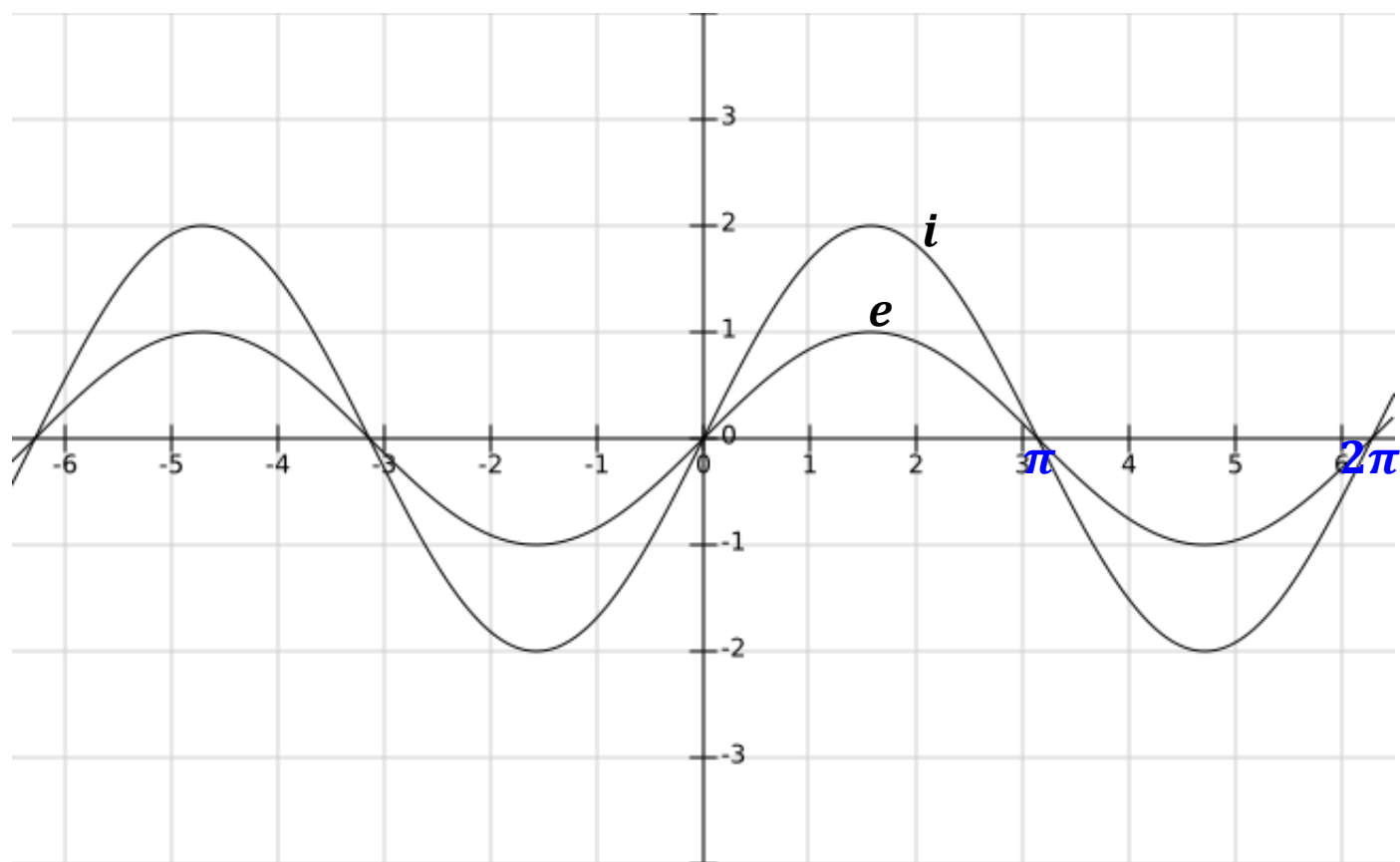
- Impedance:

$$Z_R = \frac{e}{i} = \frac{E \sin \omega t}{I \sin \omega t} = \frac{E}{I} = R$$

- Phase angle

$$\phi = 0$$



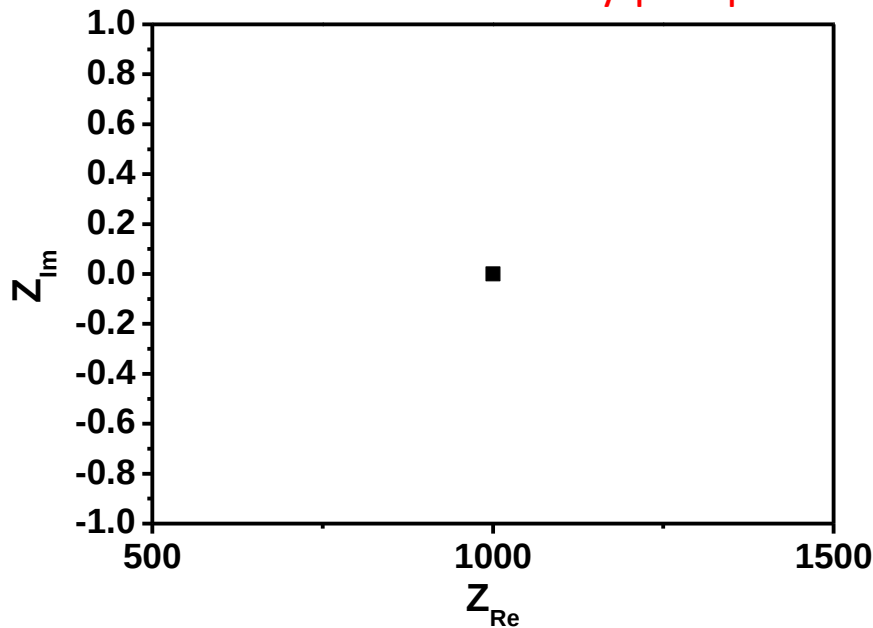


Plot of R

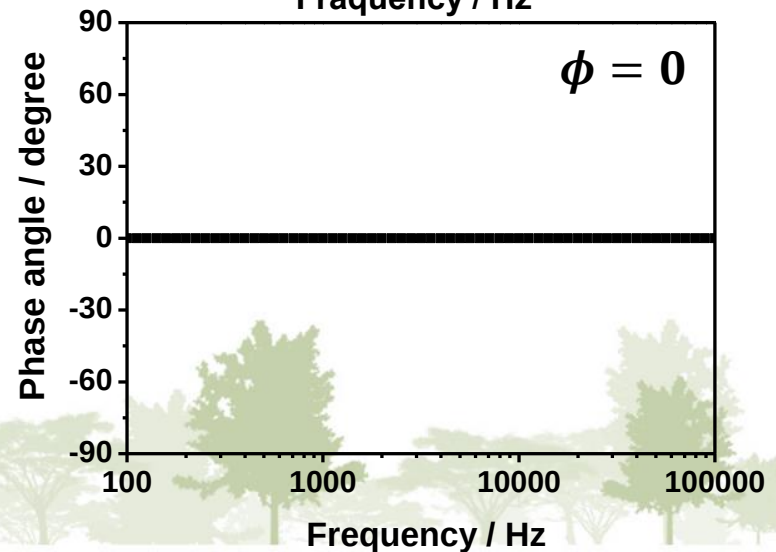
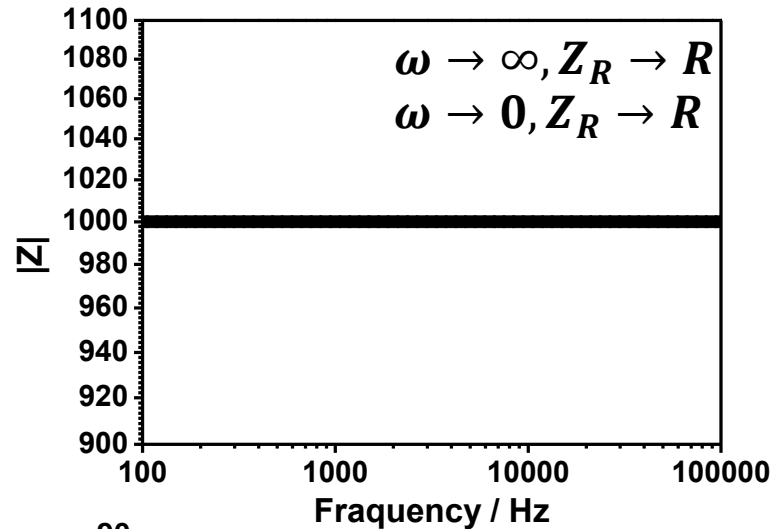
$$\text{Impedance: } Z_R = \frac{e}{i} = \frac{E \sin \omega t}{I \sin \omega t} = \frac{E}{I} = R, \text{ Phase: } \phi = 0$$

If $R = 1000 \Omega$

Nyquist plot



Bode plots



Capacitance

$$Q = C \times V$$
$$\frac{dQ}{dt} = i = C \times \frac{dV}{dt}$$

- Input:

$$e = E \sin \omega t$$

- Response:

$$i = C \times \frac{de}{dt} = C \times E \times \omega \times \cos \omega t = EC\omega \sin(\omega t + \frac{\pi}{2})$$

- Impedance:

$$Z_c = \frac{e}{i} = \frac{E \sin \omega t}{C \times E \times \omega \times \cos \omega t} = \frac{1}{\omega C} \frac{\sin \omega t}{\sin(\omega t + \frac{\pi}{2})}$$

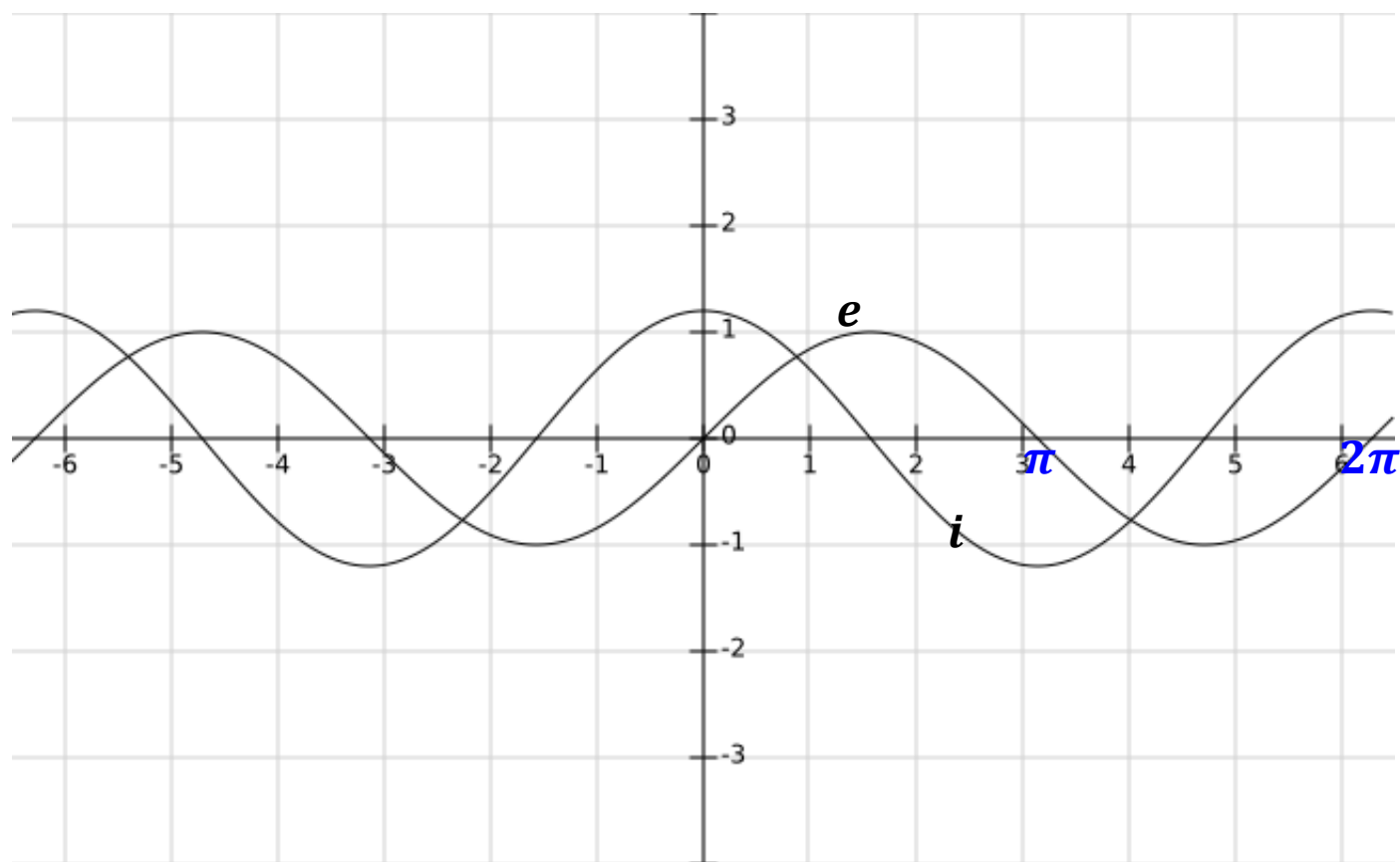
- This means that the ratio of AC voltage amplitude to AC current amplitude is $1/\omega C$ and the AC current **leads** the AC voltage by **90 degrees**. Thus, phase angle is:

$$\phi = \frac{\pi}{2}$$

- By applying Euler's formula:

$$Z_c = \frac{1}{\omega C} e^{-j\frac{\pi}{2}} = -j \frac{1}{\omega C} = \frac{1}{j\omega C}$$





Insight of Capacitor

- Input:

$$e = E \sin \omega t$$

- Response:

$$i = EC\omega \sin(\omega t + \frac{\pi}{2})$$

- Impedance:

$$Z_C = \frac{1}{\omega C} e^{-j\frac{\pi}{2}} = -j \frac{1}{\omega C} = \frac{1}{j\omega C}$$

→ $\left\{ \begin{array}{l} \omega \rightarrow \infty, Z_C \rightarrow 0 \text{ Short circuit} \\ \omega \rightarrow 0, Z_C \rightarrow \infty \text{ Open circuit} \end{array} \right.$

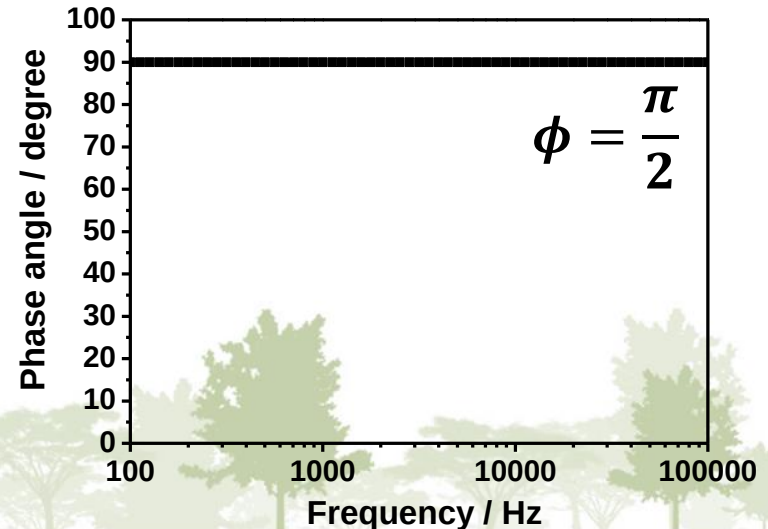
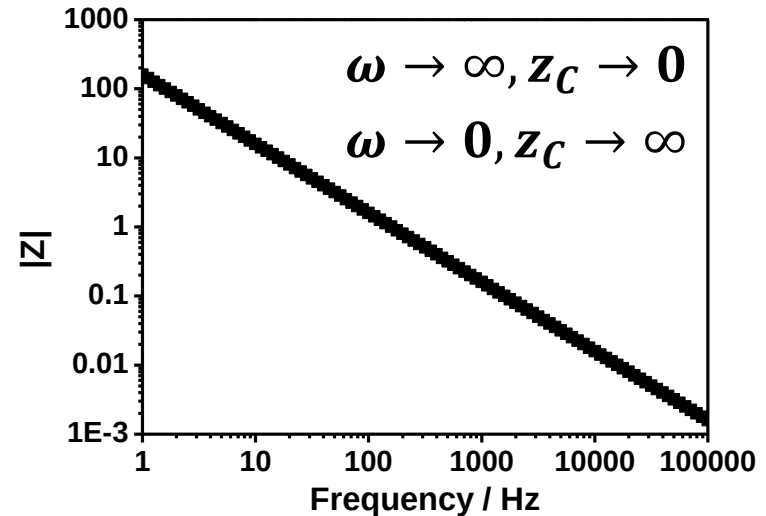
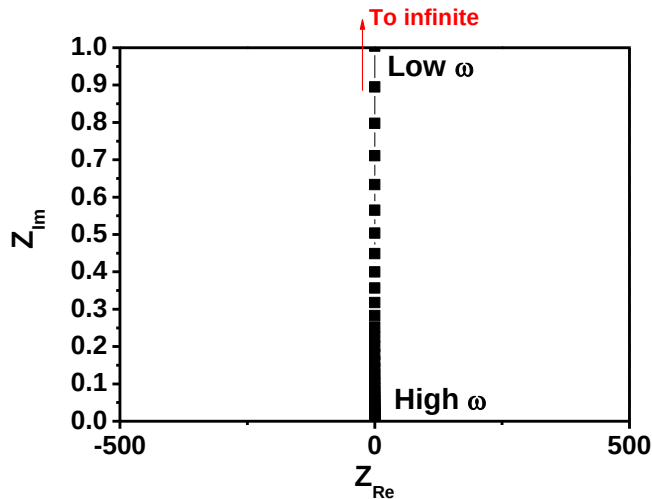
Plot of C

Impedance: $Z = -j \frac{1}{\omega C}$, phase: $\phi = \frac{\pi}{2}$

Bode plots

If $C = 0.001 \text{ F}$

Nyquist plot



Inductor

- Input:

$$i = I \sin \omega t$$

$$V = L \times \frac{dI}{dt}$$

- Response:

$$e = L \times \frac{di}{dt} = L \times I \times \omega \times \cos \omega t = IL\omega \sin(\omega t + \frac{\pi}{2})$$

- Impedance:

$$Z_L = \frac{e}{i} = \frac{I\omega L \sin(\omega t + \frac{\pi}{2})}{I \sin \omega t} = \omega L \frac{\sin(\omega t + \frac{\pi}{2})}{\sin \omega t}$$

- This means that the ratio of AC voltage amplitude to AC current amplitude is ωL and the AC current **delays** the AC voltage by **90 degrees**. Thus, phase angle is:

$$\phi = -\frac{\pi}{2}$$

- By applying Euler's formula:

$$Z_I = \omega L e^{j\frac{\pi}{2}} = j\omega L$$

Series and Parallel Z

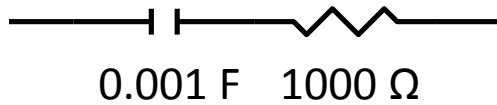
- For impedances in series:

$$\triangleright Z = Z_1 + Z_2 + \dots$$

- For impedances in parallel:

$$\triangleright Z^{-1} = Z_1^{-1} + Z_2^{-1} + \dots$$

Plot of Series RC

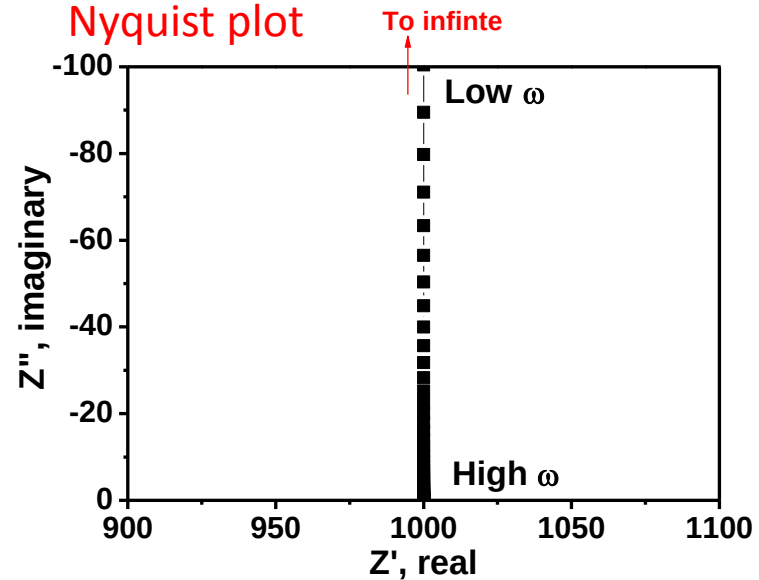


$$Z = Z_R + Z_C$$

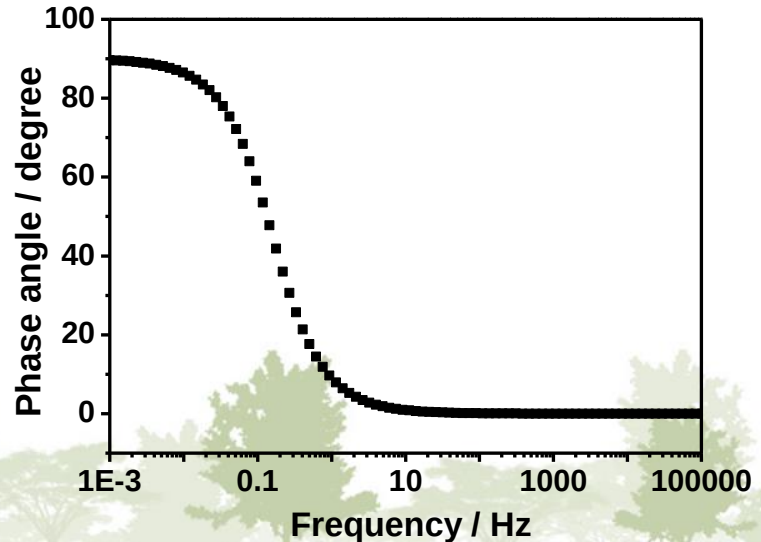
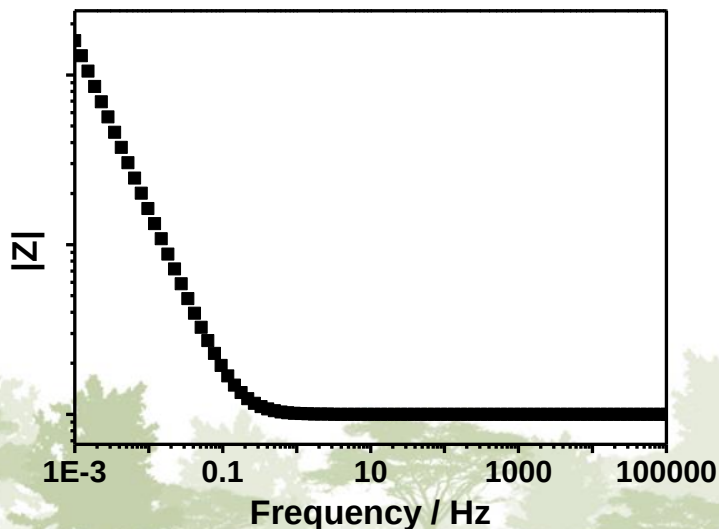
$$z(\omega) = R - j \frac{1}{\omega C} = Z_{Re} - jZ_{Im}$$

$$|Z| = \sqrt{|Z_{Re}|^2 + |Z_{Im}|^2} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

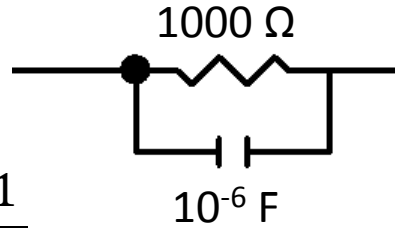
$$\tan \phi = \frac{Z_{Im}}{Z_{Re}} = \frac{1}{\omega RC}$$



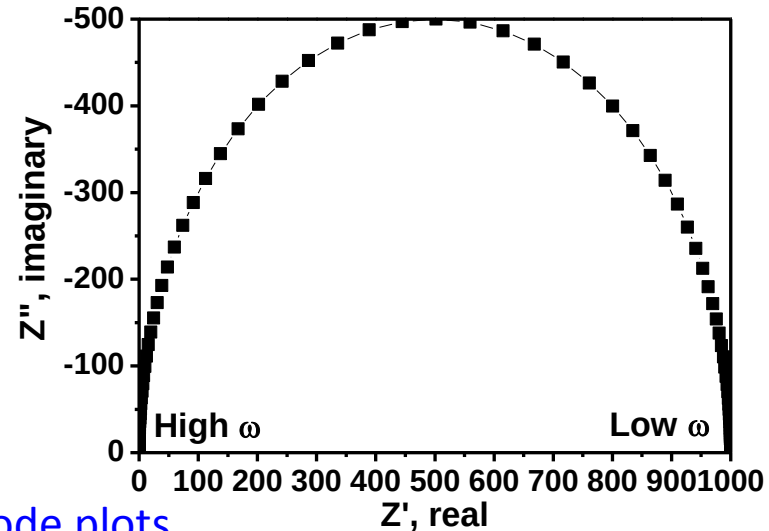
Bode plots



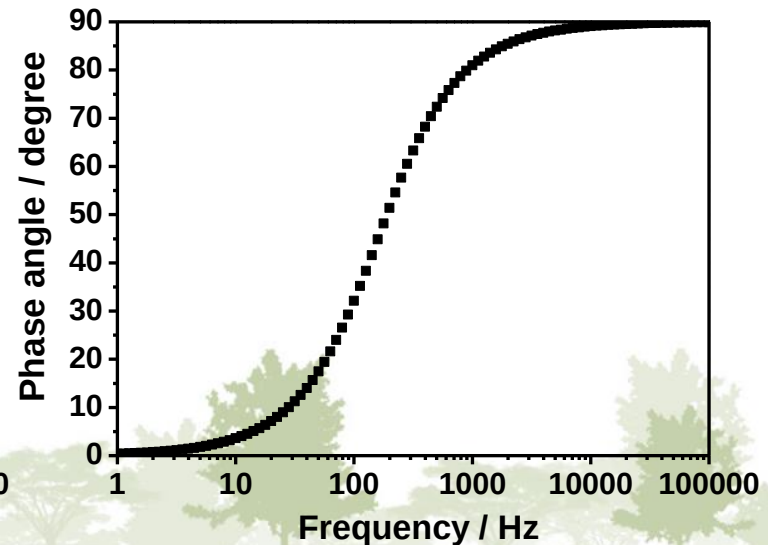
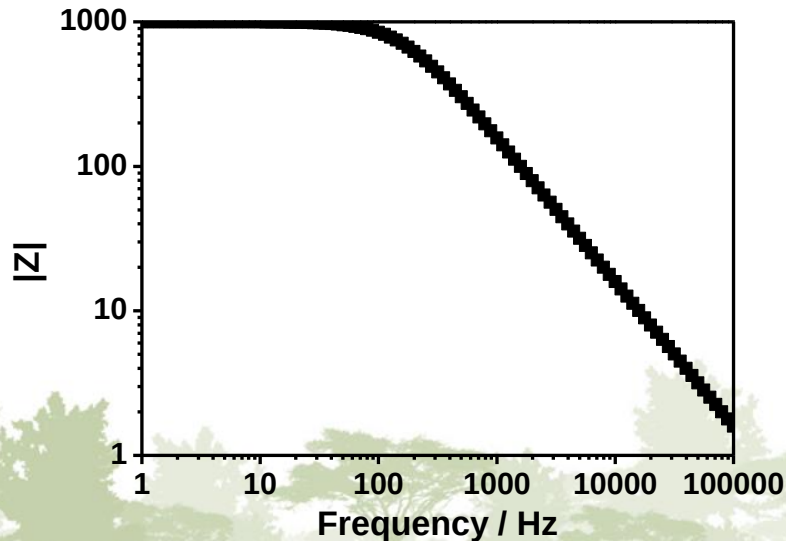
Plot of Parallel RC



Nyquist plot



Bode plots



$$\frac{1}{Z} = \frac{1}{Z_R} + \frac{1}{Z_C}$$

$$Z = \frac{Z_R Z_C}{Z_R + Z_C} = \frac{1}{\frac{1}{R_{ct}} + j\omega C_d} = R \frac{1}{1 + j\omega R_{ct} C_d}$$

$$Z = \frac{R_{ct}}{1 + \omega^2 C_d^2 R_{ct}^2} - j \left(\frac{\omega C_d R_{ct}^2}{1 + \omega^2 C_d^2 R_{ct}^2} \right)$$

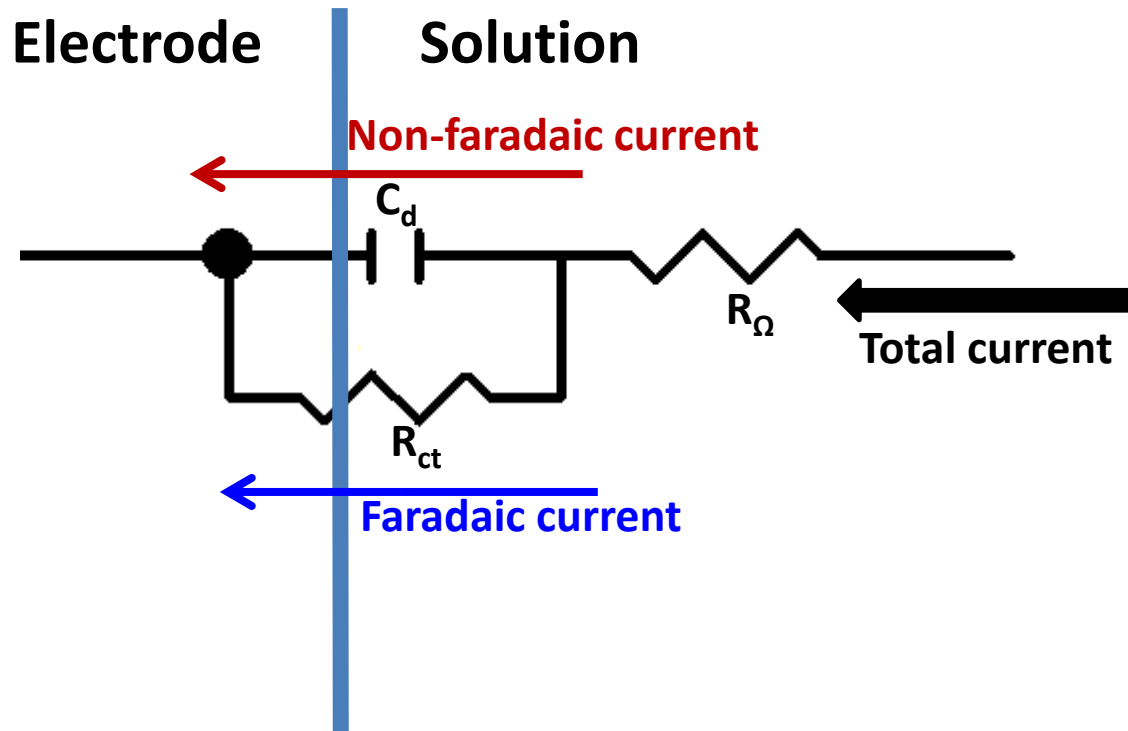
$$\tan \phi = \omega C_d R_{ct}$$

Fundamental Concept

- **Total current =**
 - Faradaic current + non-Faradaic current
- **Faradaic resistance =**
 - Charge-transfer resistor (*kinetics*) + Mass-transfer resistor (*Mass-transfer*)
- **Non-faradaic resistance =**
 - Double-layer capacitor

Kinetic Control or Charge-transfer Control

Randles Cell



R_Ω : Solution resistance

R_{ct} : Charge-transfer resistance

C_d : Capacitance

Equivalent Circuit of Randles Cell

- Total impedance:

$$Z = Z_{\Omega} + \left(\frac{1}{Z_{C_d}} + \frac{1}{Z_{R_{ct}}} \right)^{-1}$$

- Input each parameter:

$$Z = R_{\Omega} - j \left(\frac{R_{ct}}{R_{ct} C_d \omega - j} \right) = Z_{Re} - j Z_{Im}$$

$$Z = Z_{Re} - j Z_{Im} = \left(R_{\Omega} + \frac{R_{ct}}{1 + \omega^2 C_d^2 R_{ct}^2} \right) - j \left(\frac{\omega C_d R_{ct}^2}{1 + \omega^2 C_d^2 R_{ct}^2} \right)$$

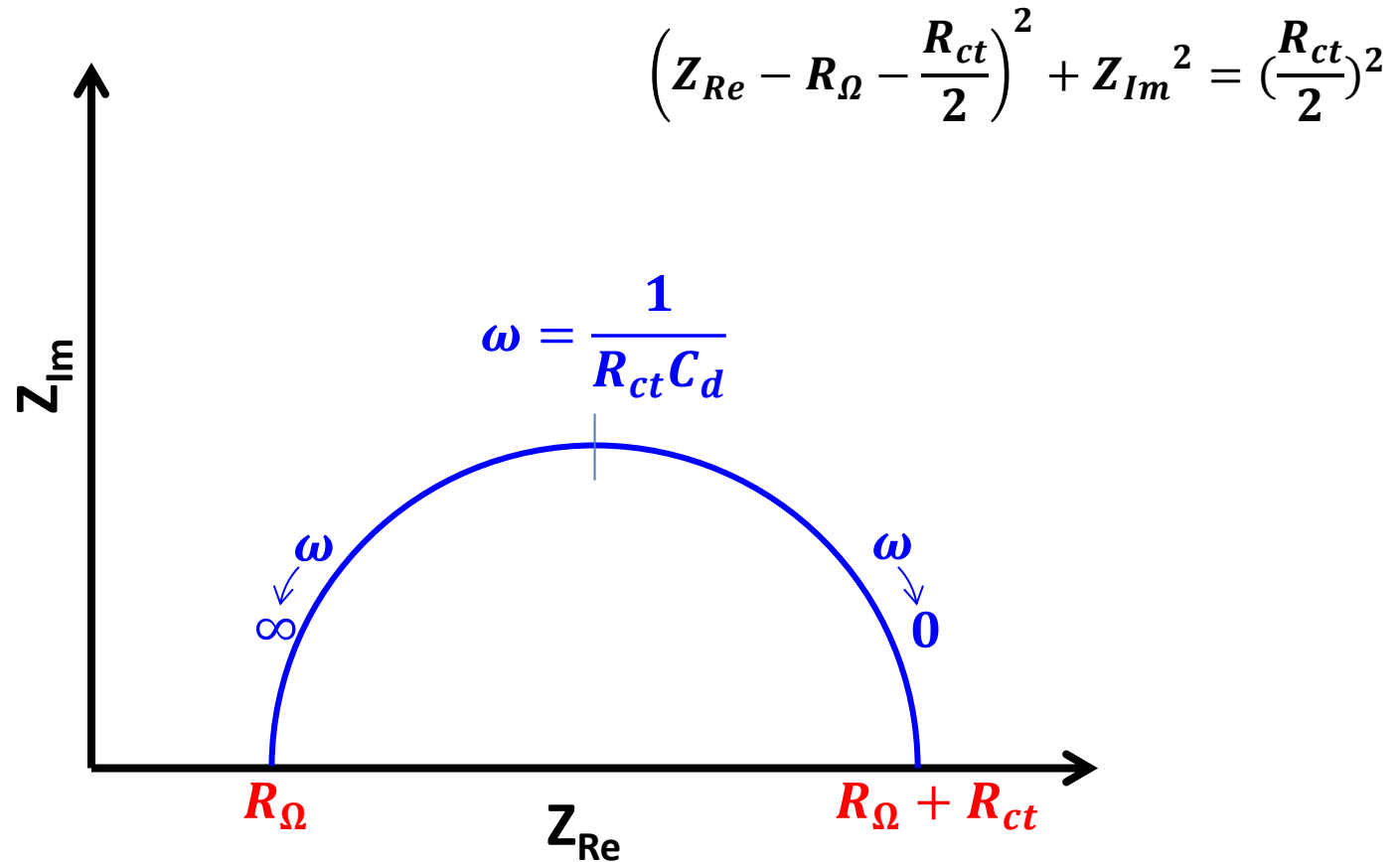
- Arrange the above equation into :

$$\left(Z_{Re} - R_{\Omega} - \frac{R_{ct}}{2} \right)^2 + Z_{Im}^2 = \left(\frac{R_{ct}}{2} \right)^2 \quad \boxed{\text{Semi-circle}}$$

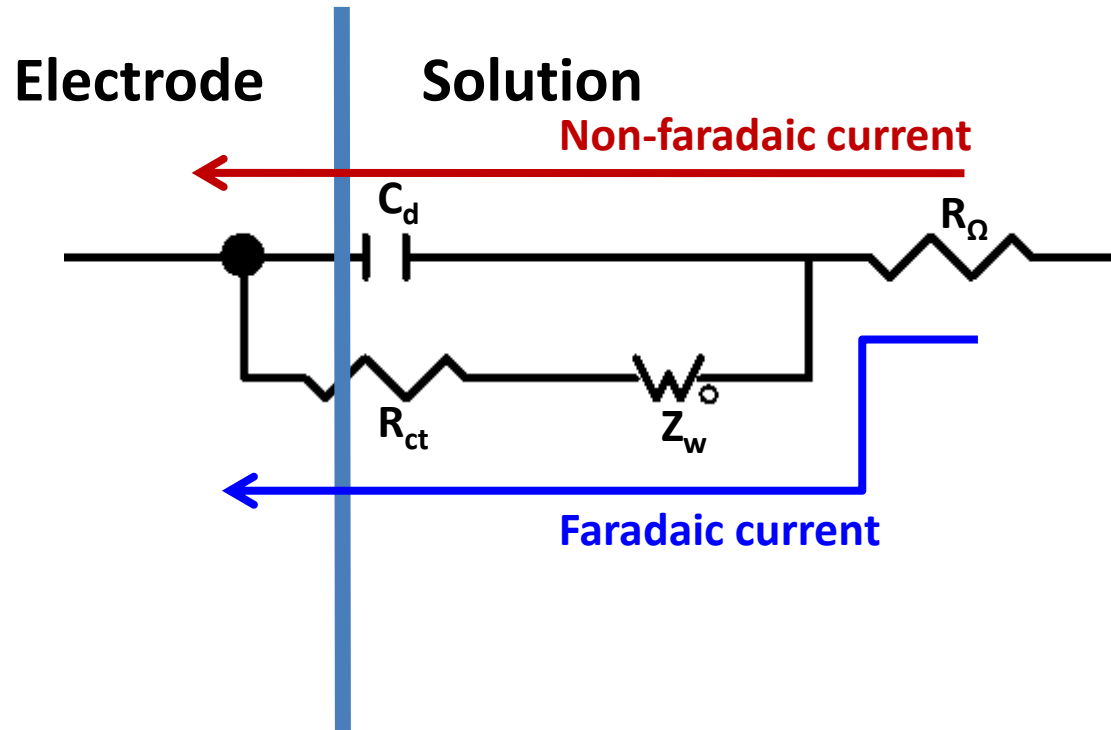
$$R_{ct} = \frac{RT}{F i_0}$$



Nyquist Plot of Randles Model



Total Impedance: Considering of Mass Transfer and Charge Transfer



R_Ω : Solution resistance
 R_{ct} : Charge-transfer resistance
 C_d : Capacitance
 Z_w : Warburg impedance

Equivalent Circuit of Mixed Control

- Total impedance:

$$Z = Z_{Re} - jZ_{Im}$$

$$Z_{Re} = R_{\Omega} + \frac{R_{ct} + \sigma\omega^{-1/2}}{(C_d\sigma\omega^{1/2} + 1)^2 + \omega^2 C_d^2 (R_{ct} + \sigma\omega^{-1/2})^2}$$

$$Z_{Im} = \frac{\omega C_d (R_{ct} + \sigma\omega^{-1/2})^2 + \sigma\omega^{-1/2} (\omega^{1/2} C_d \sigma + 1)}{(C_d\sigma\omega^{1/2} + 1)^2 + \omega^2 C_d^2 (R_{ct} + \sigma\omega^{-1/2})^2}$$

$$\sigma = \frac{RT}{n^2 F^2 A \sqrt{2}} \left(\frac{1}{D_o^{1/2} C_o^*} + \frac{1}{D_R^{1/2} C_R^*} \right)$$



High and Low Frequency

- At high frequency:

$$Z_{Re} = R_{\Omega} + \frac{R_{ct}}{1 + \omega^2 C_d^2 R_{ct}^2}$$

$$Z_{Im} = \frac{\omega C_d R_{ct}^2}{1 + \omega^2 C_d^2 R_{ct}^2}$$

$$\left(Z_{Re} - R_{\Omega} - \frac{R_{ct}}{2} \right)^2 + Z_{Im}^2 = \left(\frac{R_{ct}}{2} \right)^2$$

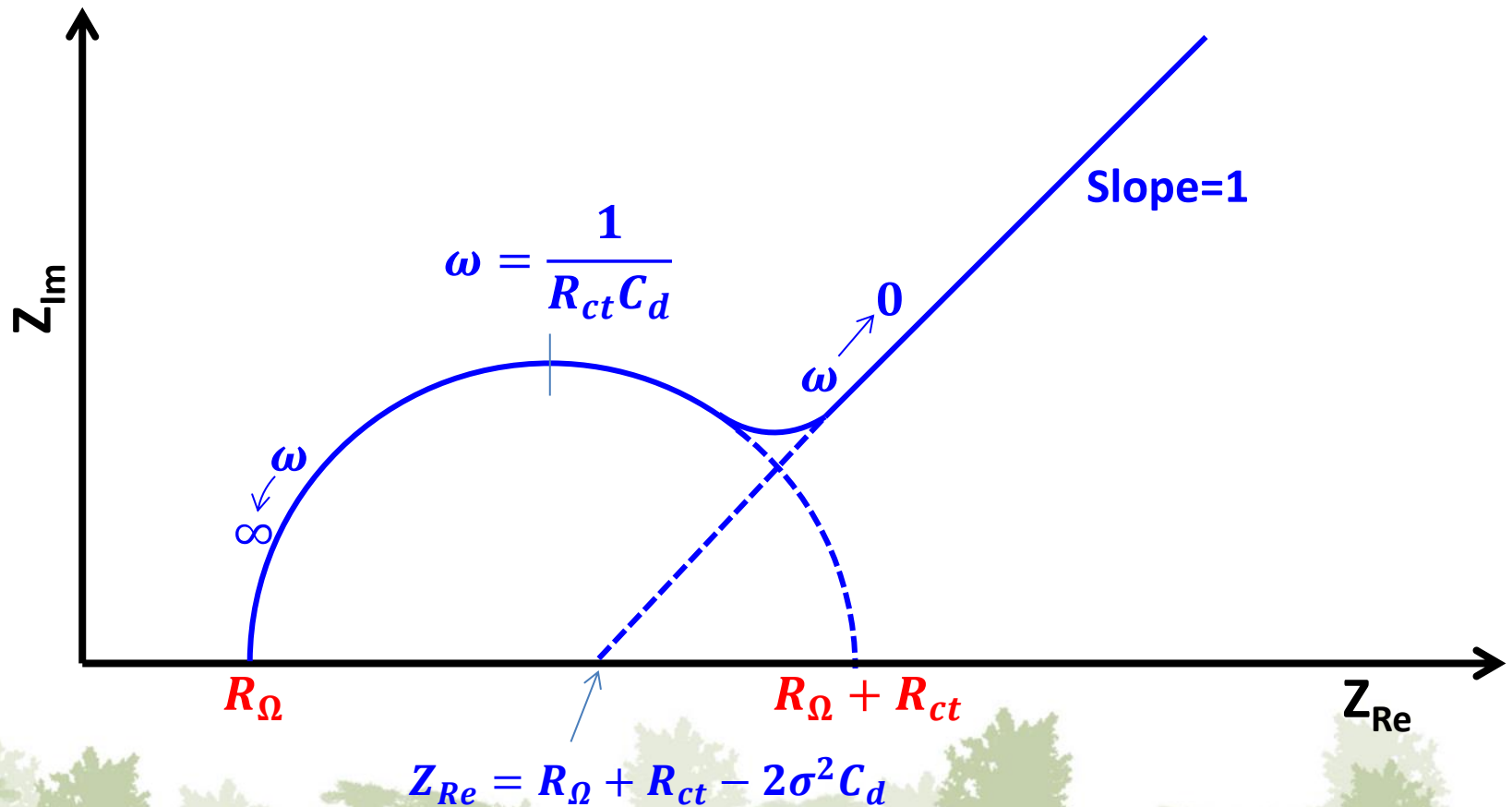
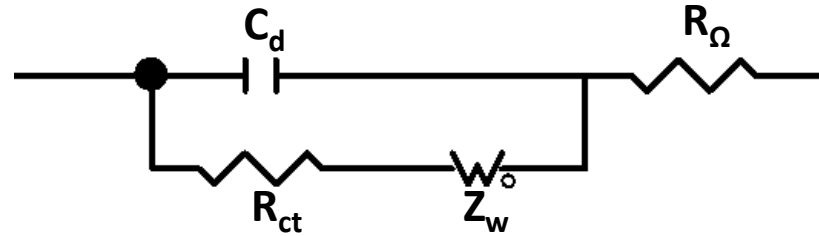
- At low frequency:

$$Z_{Re} = R_{\Omega} + R_{ct} + \sigma \omega^{-1/2}$$

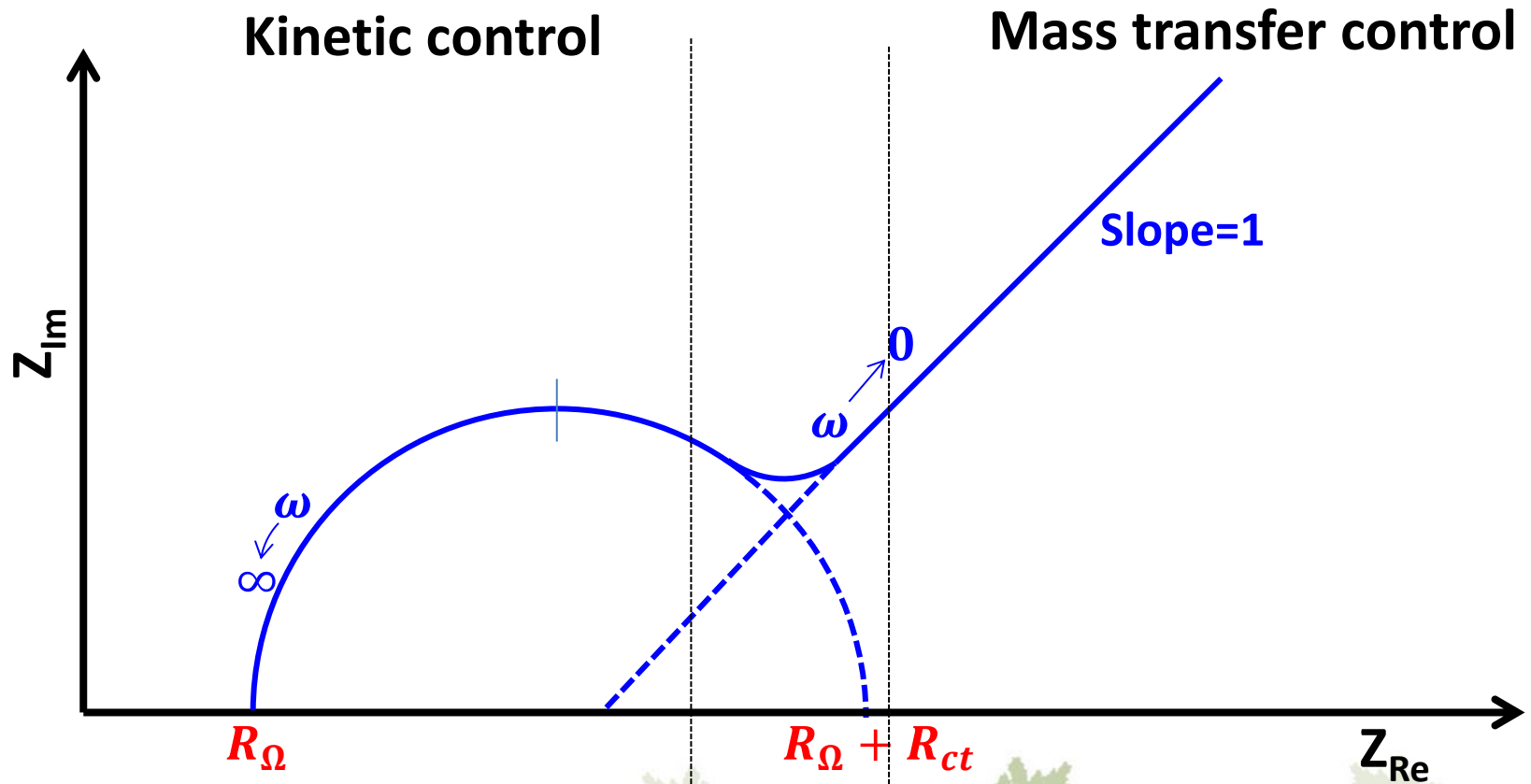
$$Z_{Im} = \sigma \omega^{-1/2} + 2\sigma^2 C_d$$

$$Z_{Im} = Z_{Re} - R_{\Omega} - R_{ct} + 2\sigma^2 C_d$$

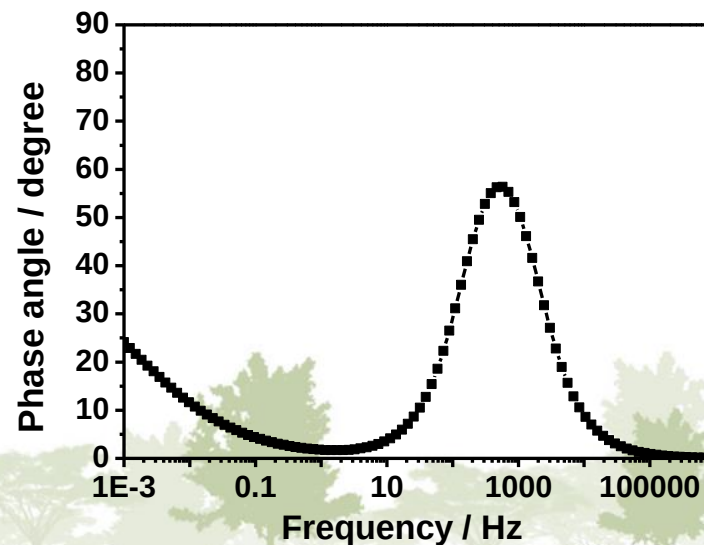
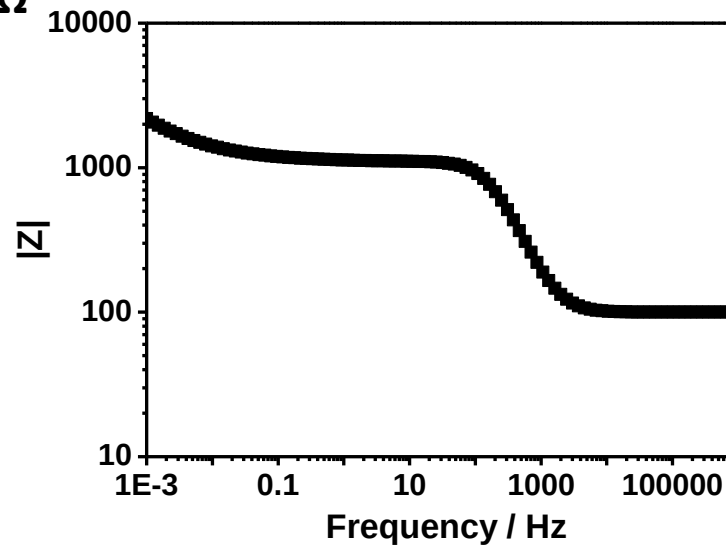
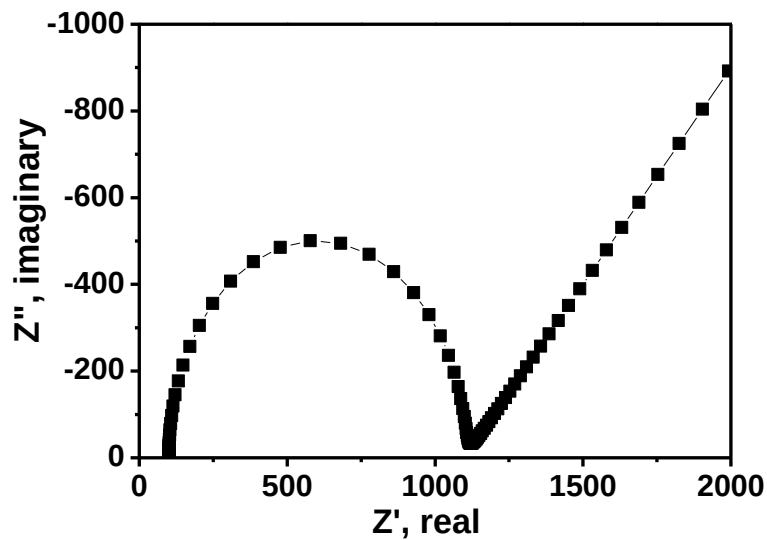
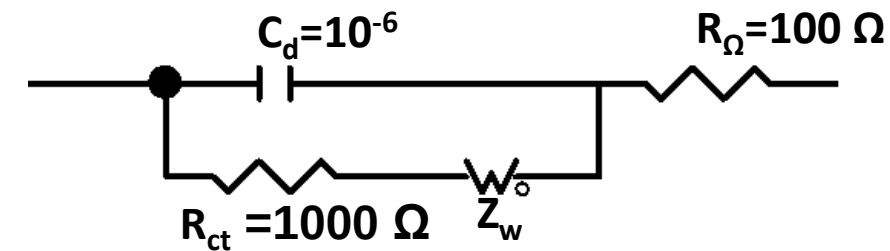
Nyquist Plot of Mixed Control



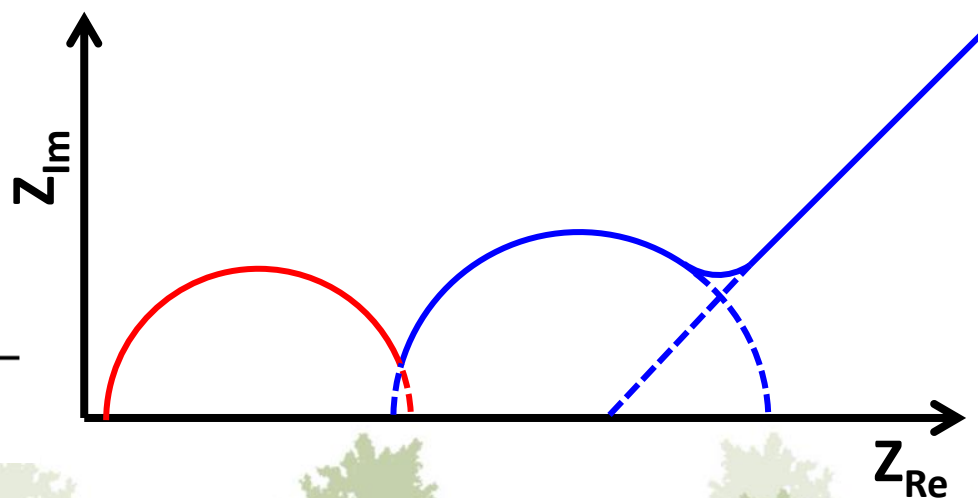
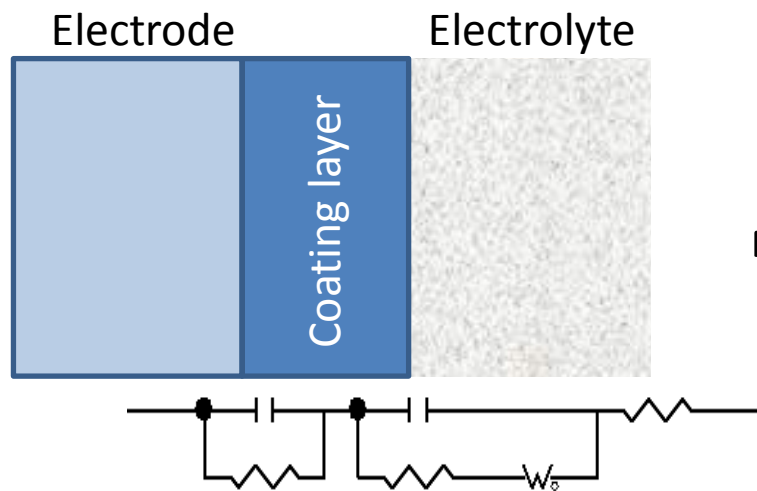
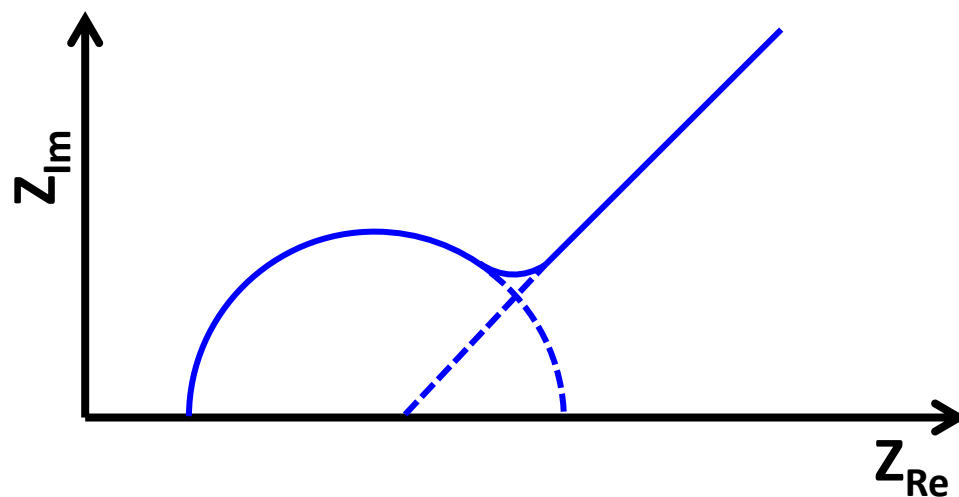
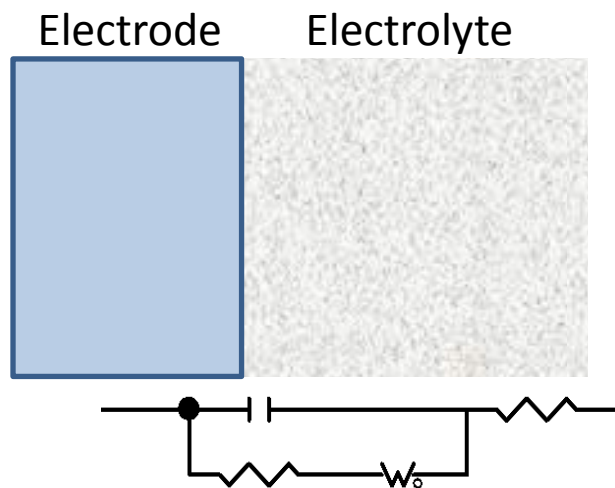
Kinetic Control and Mass Transfer Control



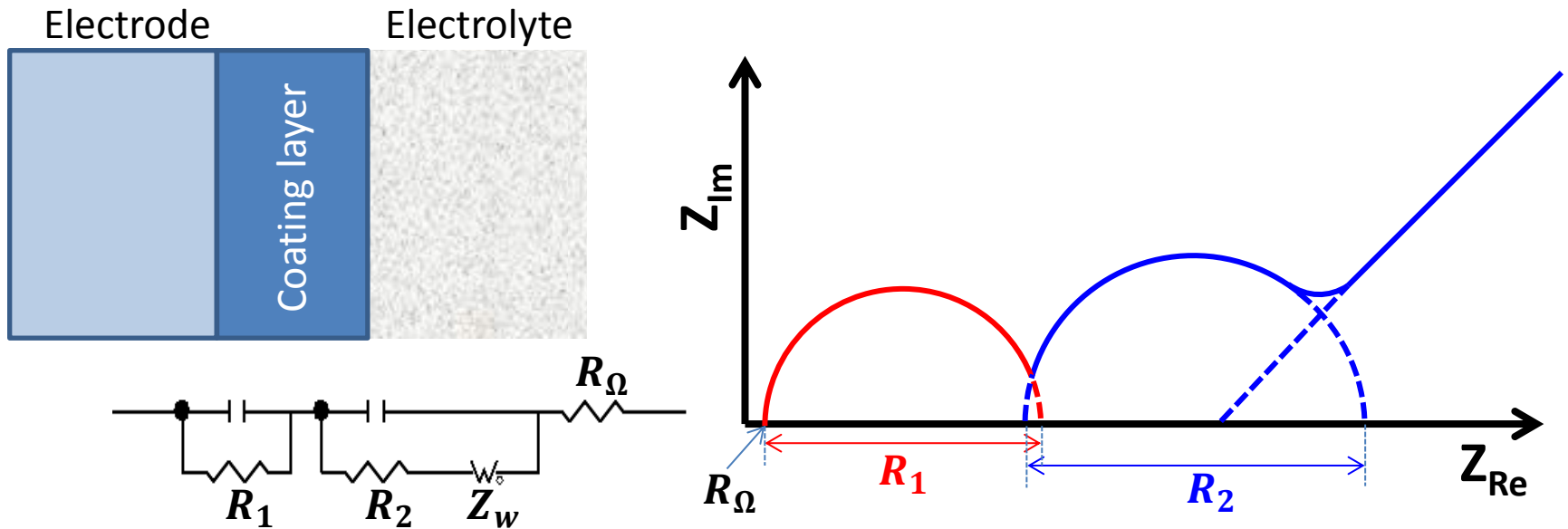
Plot of Mixed Control



General Models for Simple Cases



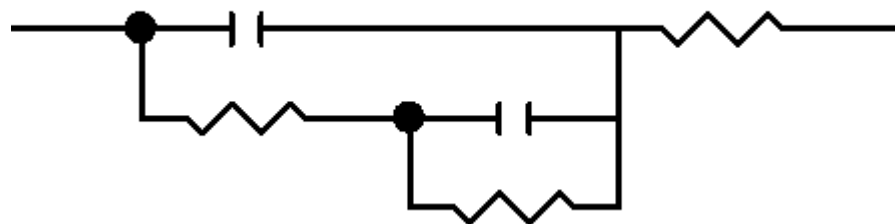
Two Parallel RC Circuits



Mathematical Equivalence of Circuits



Two resistive layers



A reaction mechanism comprising two electrochemical steps or a system consisting of a coated layer



A solid-state Schottky diode with a leakage current and deep-level electronic states

These three models look like **very different physical meaning**, but they have **the same frequency response**. Thus, additional investigations are essential to validate a model.

Measurement Setup

ZPlot - FC-OCP.zpw (1280C)

File Setup Measure Help

Icons: Folder, Save, Print, Data, Z, Plot, Sweep, Z, Help

Ctrl I: Sweep Freq	Ctrl I: Sweep DC	Ctrl I: Sweep Ampl	Ctrl I: Vs. Time
Ctrl E: Sweep Freq	Ctrl E: Sweep DC	Ctrl E: Sweep Ampl	Ctrl E: Vs. Time

Polarization:

DC Potential (Volts) vs. Open Circuit

AC Amplitude (mV)

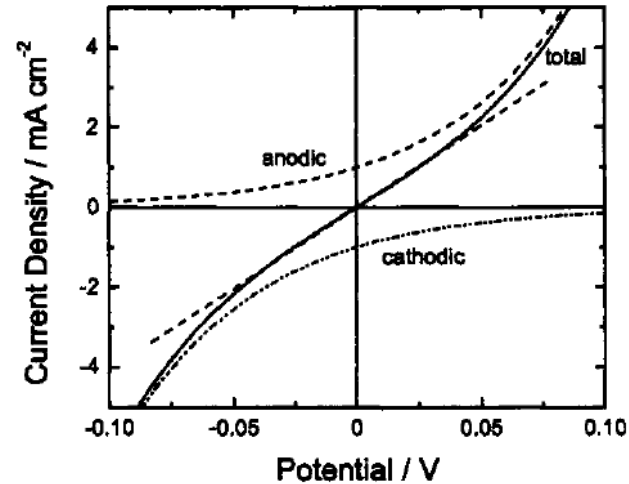
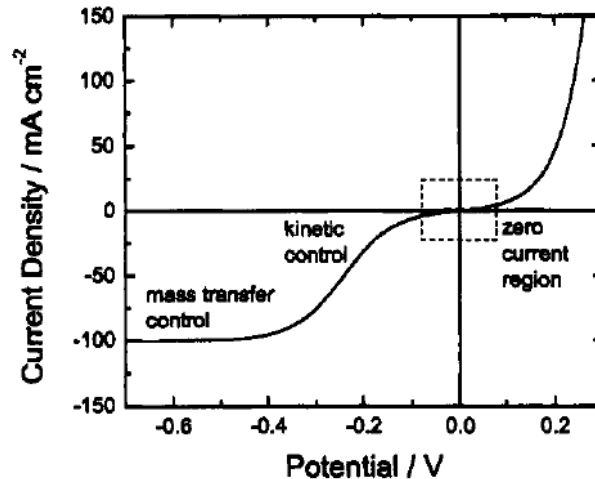
☐ **Monitor Cell Potential**
OCP (Volts): Not Available

Frequency Sweep:

Initial Frequency (Hz) ☐ Linear ☒ Logarithmic ☐ List

Final Frequency (Hz) Steps/Decade Interval

Kinetic Control



- Bulter-Volmer equation:

$$i = i_0 \left\{ \exp \left[\frac{(1 - \alpha)nF}{RT} \eta \right] - \exp \left[-\frac{\alpha nF}{RT} \eta \right] \right\} = i_0 [\exp(b_a \eta) - \exp(-b_c \eta)]$$

$$b_a = \frac{(1 - \alpha)nF}{RT}; b_c = \frac{\alpha nF}{RT}; i_0 = nFk_0$$

$$k_0 = k_a = k_c$$

- Linear function (Linearity):

$$i = \frac{ni_0F}{RT} \eta$$

Frequency Range

- The measured frequency range should include frequencies sufficiently **large** and frequencies sufficiently **small** to reach asymptotic limits in which the **imaginary impedance tends toward zero** [1].
- In some cases, the low-frequency asymptotic behavior does not exist, such as blocking electrodes.
- The high-frequency asymptotic behavior is limited by the instrument artifact.

Blocking electrode: Due to the electrode geometry, the non-uniform current and potential distribution associated with the disk geometry influences the transient and the impedance response [1].

Characteristic Frequency

- Parallel RC

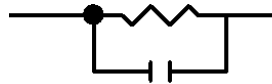
$$z = \frac{1}{\frac{1}{R_{ct}} + j\omega C_d} = \frac{R_{ct}}{1 + j\omega R_{ct} C_d}$$

- Characteristic frequency

$$\omega_c = 2\pi f = \frac{1}{R_{ct} C_d}$$

$$z_c = R \frac{1}{1 + j} = \frac{R}{\sqrt{2}} e^{-j\frac{\pi}{4}}$$

- At the characteristic frequency, the current signal lags the potential input by **45 degrees**.



- Faradic current

$$i_f = nFk_a e^{b_a V} - nFk_c e^{-b_c V}$$

- The input signal can be represented by:

$$V = \bar{V} + \Delta V \cos \omega t = \bar{V} + \Delta V \cos(2\pi f t)$$

- A typical electrochemical reaction is followed by:

$$C_d = 31 \frac{\mu F}{cm^2}, nFk_a = nFk_c = 0.5 \frac{mA}{cm^2}$$

$$b = b_a = b_c = 19.5 V^{-1}$$

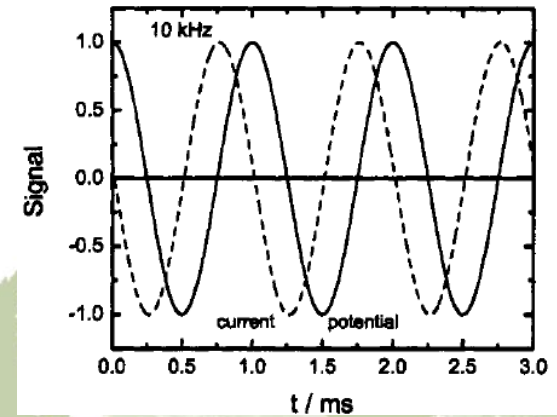
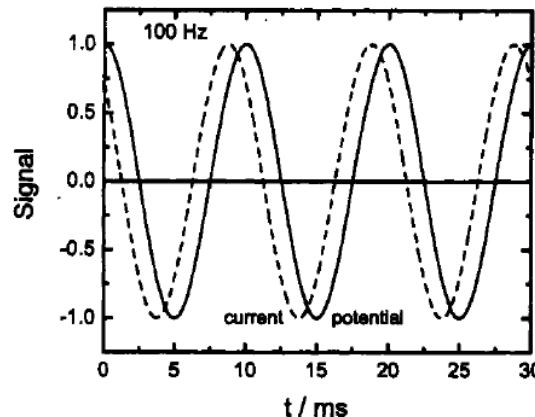
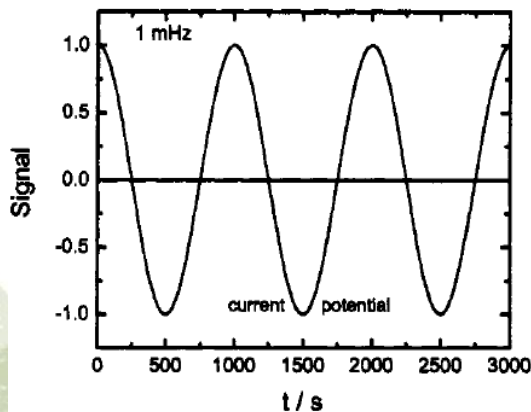
$$\bar{V} = 0 V, \Delta V = 1 mV$$

- The charge-transfer resistance is:

$$R_{ct} = \frac{1}{(b_a nFk_a e^{b_a \Delta V} + b_c nFk_c e^{-b_c \Delta V})} = 51.28 \Omega cm^2$$

- Thus, the characteristic frequency is

$$f_c = \frac{1}{2\pi R_{ct} C_d} = \mathbf{100.16 Hz}$$



Linearity and Amplitude

$$V = \bar{V} + \Delta V \cos \omega t = \bar{V} + \Delta V \cos(2\pi f t)$$

- The suitable amplitude (ΔV) is the desire to minimize nonlinear response and the desire to minimize noise in the impedance response.
- **A small amplitude can induce low nonlinear response.** If the system has very non-linear current-potential curve, a much smaller amplitude is needed.
- **A high amplitude can reduce lower noise.** If the system exhibit linear current-potential curve, a much larger amplitude can be used.
- Considering of a typical electrochemical reaction, the current density response to a potential perturbation by Taylor's expansion:

$$i(t) = i_0 \left[\overbrace{\left(1 + \frac{b^2 \Delta V^2}{4} \right)}^{\text{DC current}} + \overbrace{\left(b \Delta V + \frac{3b^3 \Delta V^3}{24} \right)}^{\text{First harmonic}} \cos(2\pi f t) + \dots \right]$$

$$i_0 = K e^{b\bar{V}}$$

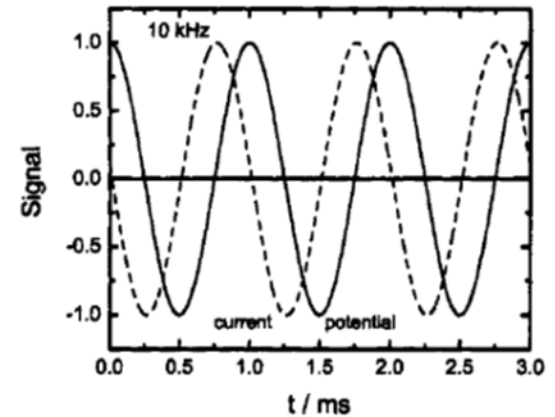
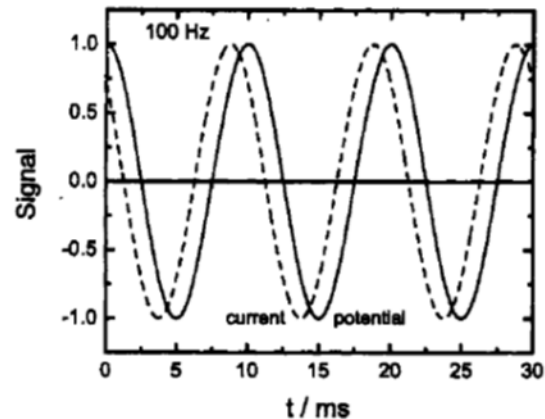
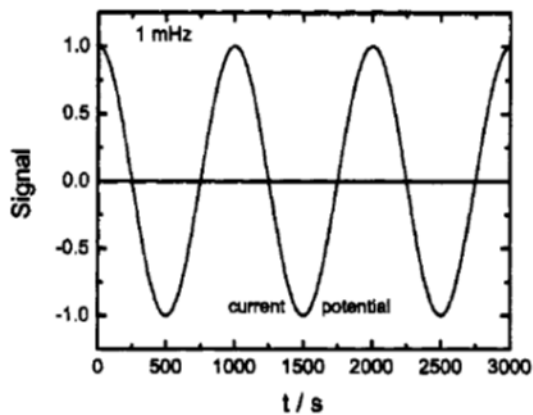
- For $b\Delta V \sim 0.2$, the variation of DC current is less than 1% and the variation of the first harmonic is smaller than 0.22%.

Current Density Response by Different Amplitude

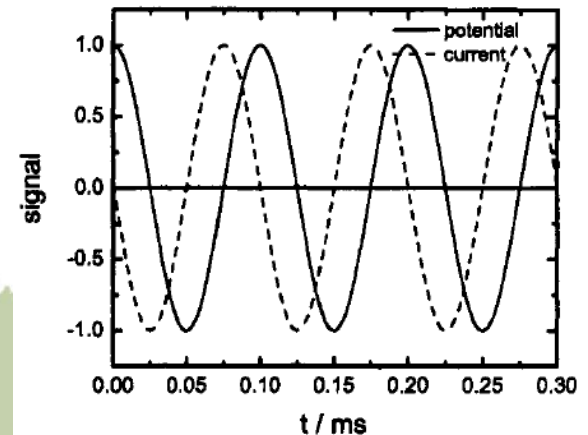
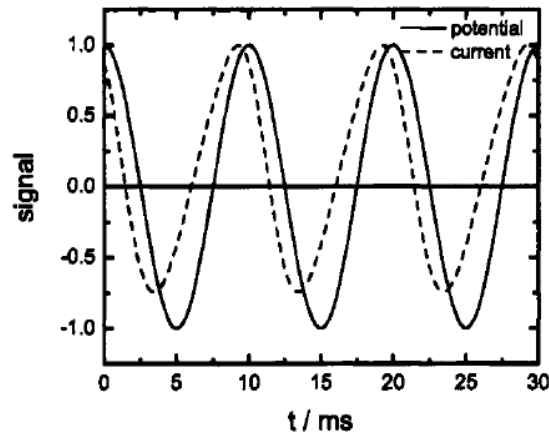
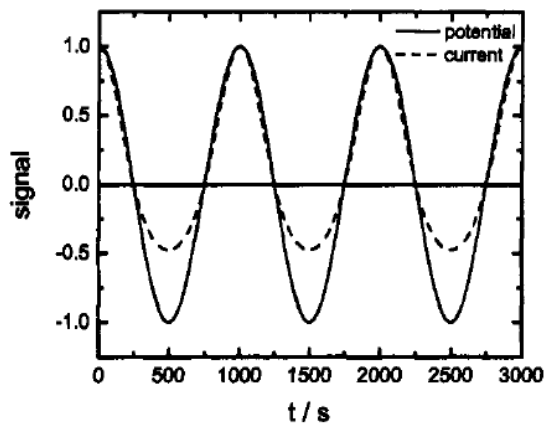
$$C_d = 31 \frac{\mu F}{cm^2}, nFk_a = nFk_c = 0.5 \frac{mA}{cm^2}, b = b_a = b_c = 19.5 V^{-1}, \bar{V} = 0 V$$

- Two different amplitudes, 1 mV and 40 mV, are carried out, so the results of $b\Delta V$ are 0.0195 and 0.78, respectively.

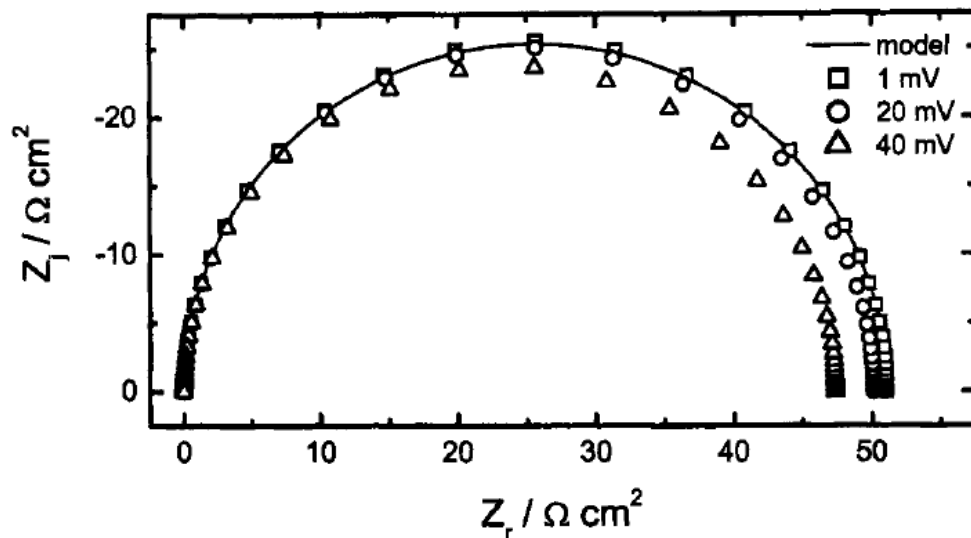
$\Delta V = 1 mV$



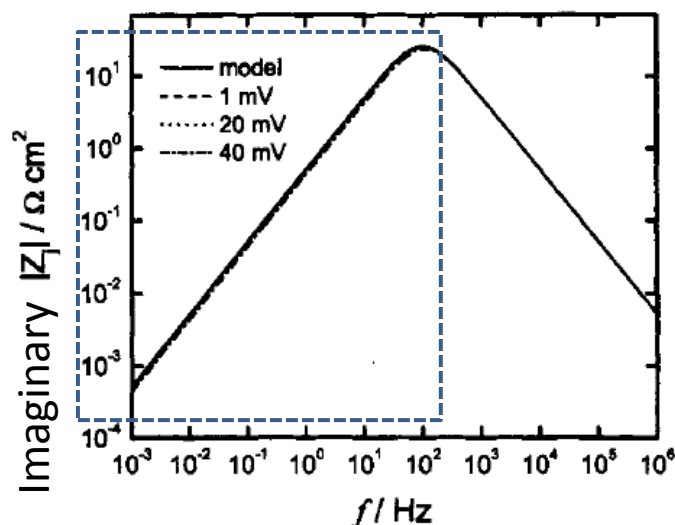
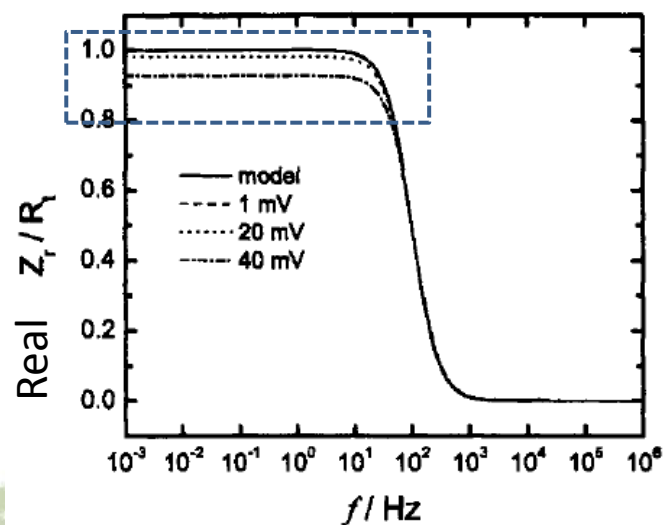
$\Delta V = 40 mV$ Not symmetric about zero at low frequency



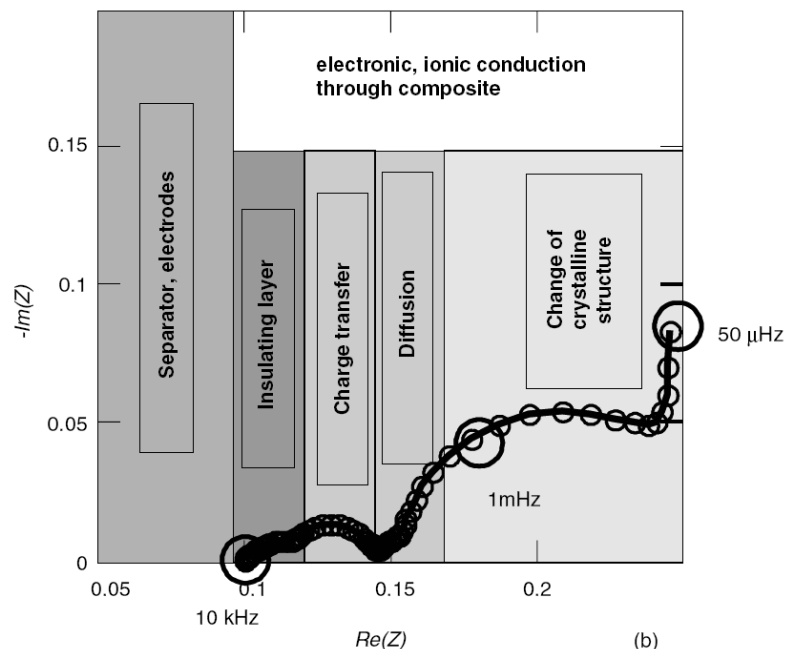
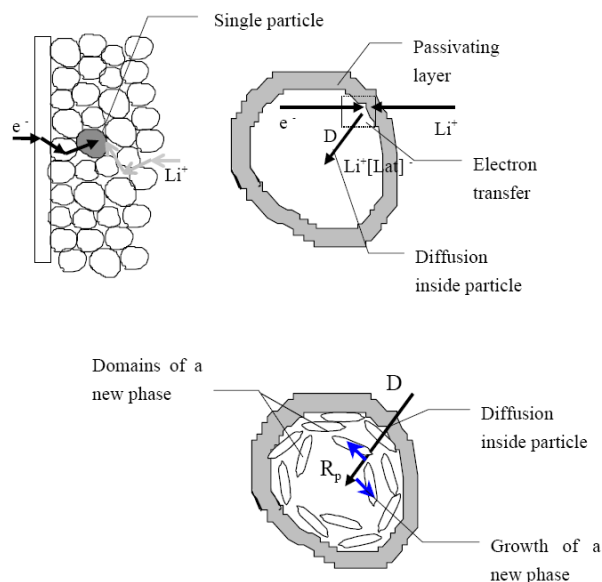
Cont'd



- The large amplitude cause an error in the impedance response, which arises from the nonlinear behavior of the Faradaic current.
- This effect is very clearly seen in the real part of the impedance at the low-frequency response.
- While $b\Delta V$ is 0.2, the error at the low-frequency response is 0.5%. Thus, for the typical electrochemical reaction, the amplitude is below 20 mV.



Example of Lithium-ion Battery

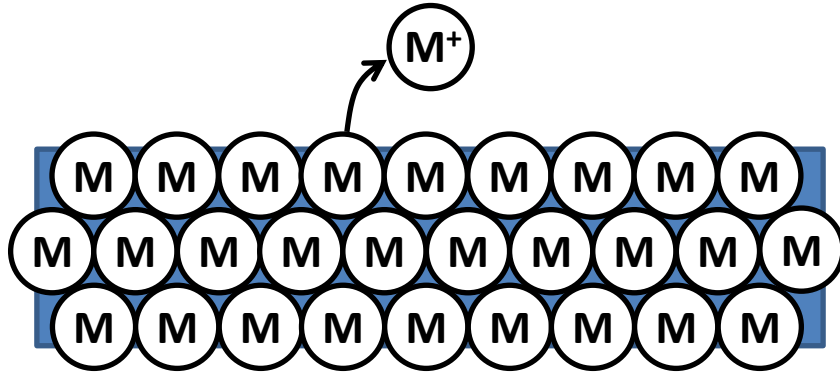


- **Higher than 10 kHz:** The inductance of wires and electrode winding.
- **Around 10 kHz:** Conduction through the electrolyte and porous separator, and conduction through wire.
- **Around 1 kHz:** Charge transfer involves the resistance of an insulating layer and activated electron transfer resistance on the electronic/ionic conduction boundary.
- **Around 100 Hz:** The electronic conduction through the particles and ionic conduction through the electrolyte in cavities between particles
- **Lower than 1 Hz:** The formation of new crystalline structures.

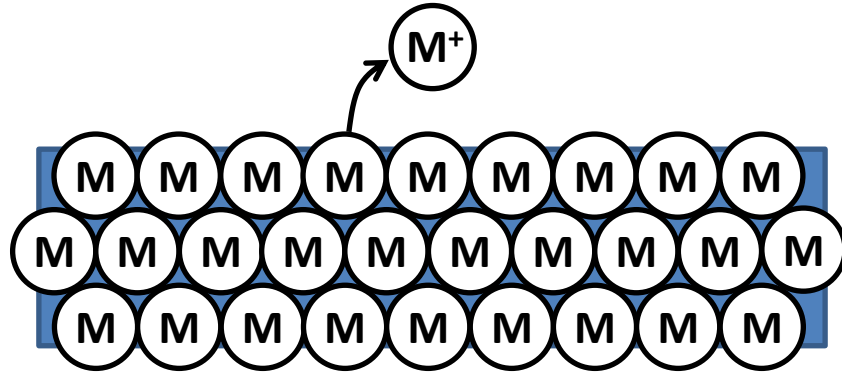
[1] E. Barsoukov, J.H. Kim, D.H. Kim, K.S. Hwang, C.O. Yoon, H. Lee, J. New Mater. Electrochem. Syst., 3 (2000) 301-308.

[2] E. Barsoukov, J.R. Macdonald, Impedance spectroscopy : theory, experiment, and applications, Wiley-Interscience, Hoboken, N.J., 2005.

Metal Dissolution



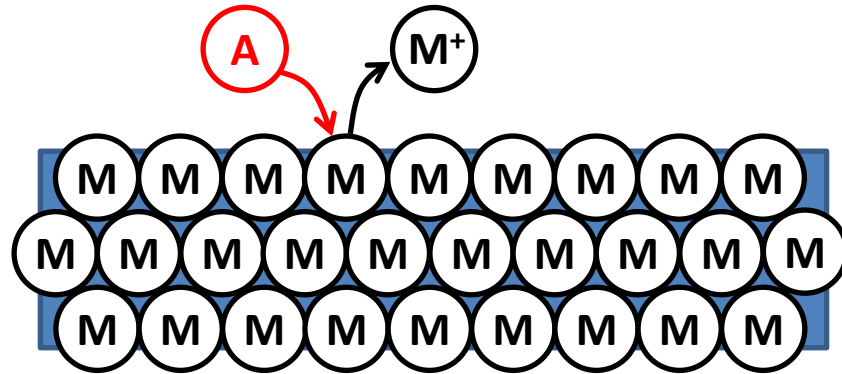
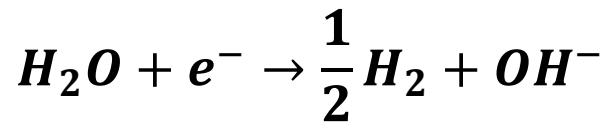
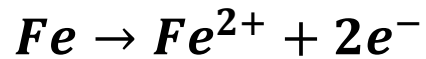
Metal Dissolution with **Mass Transfer**



Metal Dissolution by Reaction with Electrolytic Species



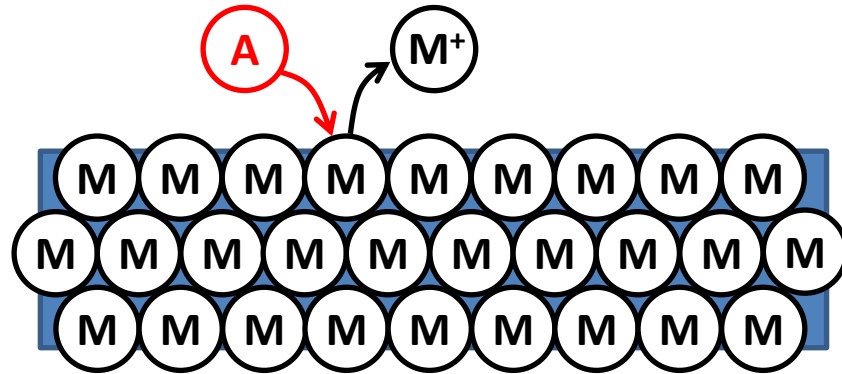
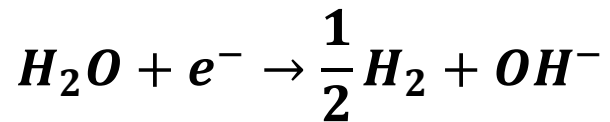
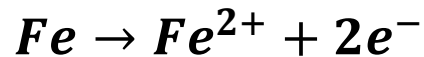
Example:



Metal Dissolution by Reaction with Electrolytic Species and **Mass Transfer**



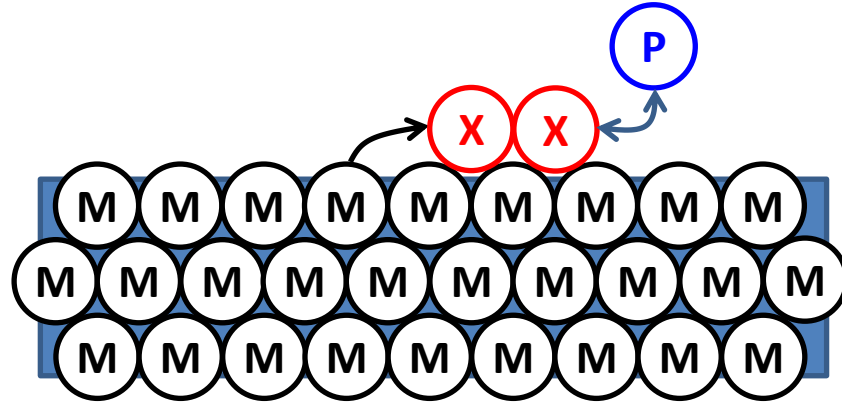
Example:



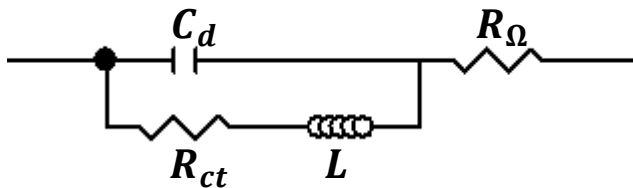
Metal Dissolution with Surface Coverage



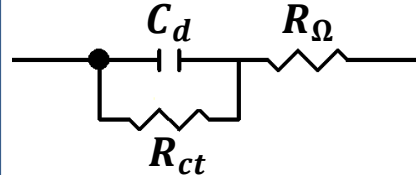
The surface coverage rates of intermediate, X , and product, Y , determine the model.



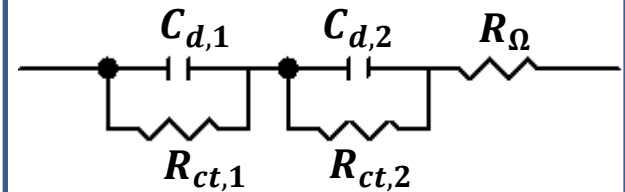
Low surface coverage
(Adsorption)



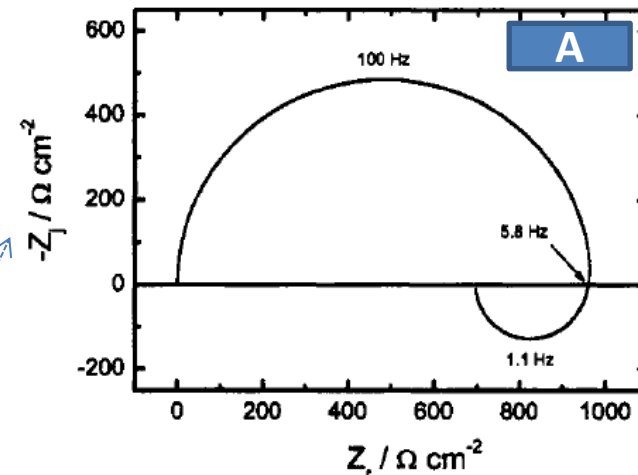
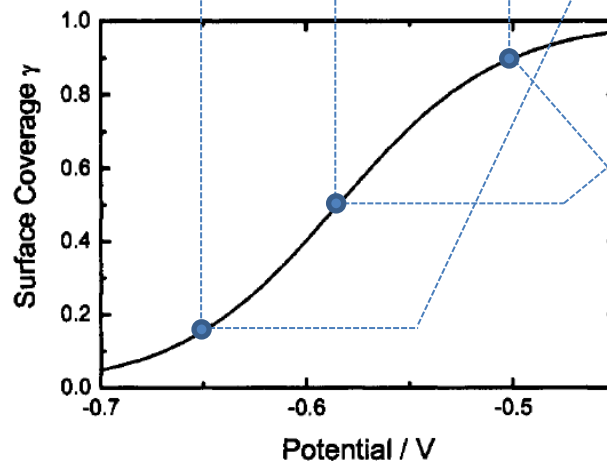
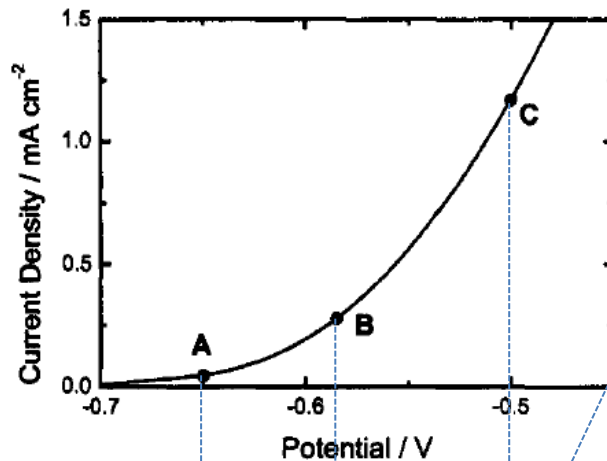
Moderate surface coverage



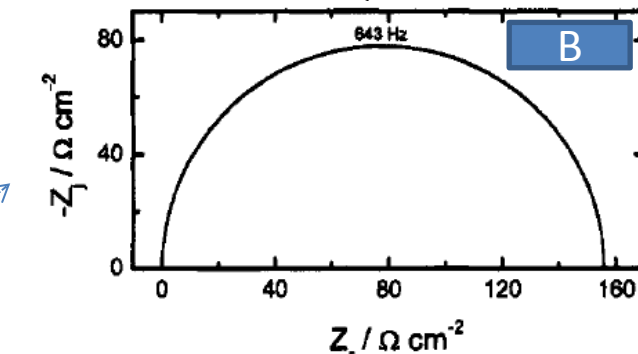
Thick surface coverage



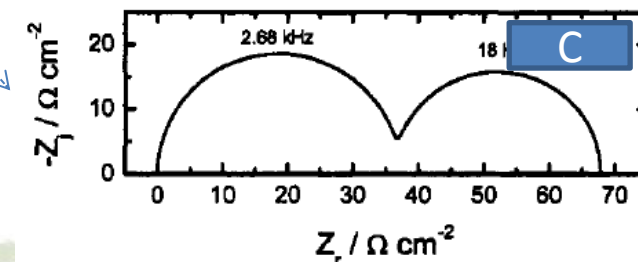
Surface Coverage and Impedance Response



$V = -0.65 \text{ V}$



$V = -0.585 \text{ V}$



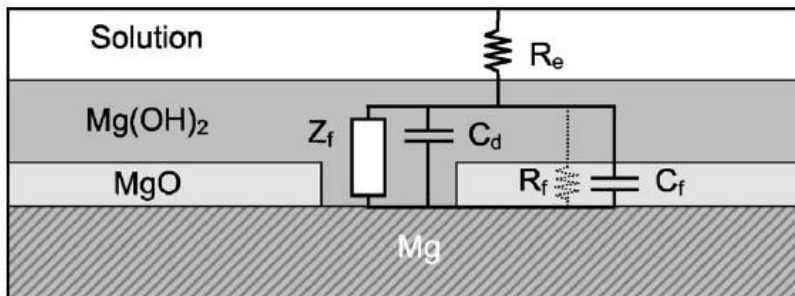
$V = -0.5 \text{ V}$

$$K_M = 4F A \text{ cm}^{-2}, b_M = 36 \text{ V}^{-1}, K_X = 10^{-6} F A \text{ cm}^{-2}, b_X = 10 \text{ V}^{-1}, \Gamma = 2 \times 10^{-9} \text{ mol cm}^{-2}, C_d = 20 \mu F \text{ cm}^{-2}$$

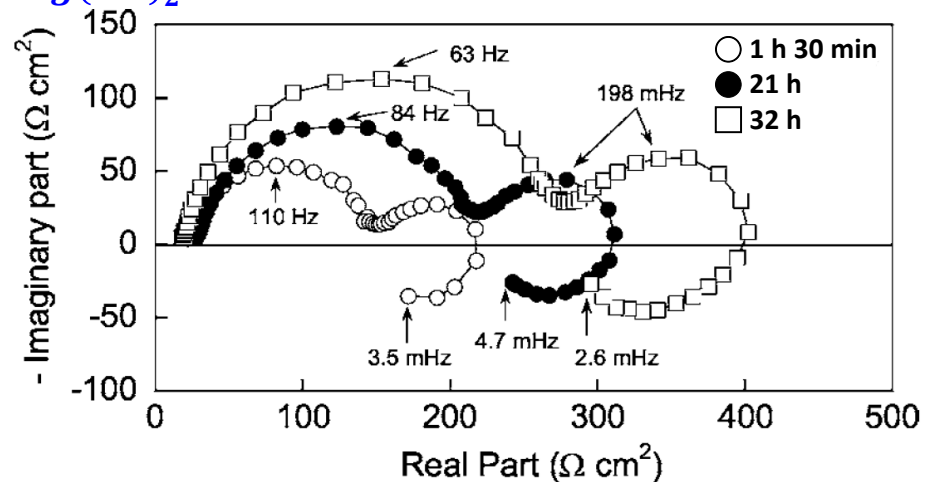
Mg Corrosion in Na₂SO₄ Solution

Mg corrosion in Na₂SO₄ solution ($E \rightarrow C \rightarrow C$)

- Anodic partial reaction: $2Mg \rightarrow 2Mg^+ + 2e^-$
- Chemical reaction: $2Mg^+ + 2H_2O \rightarrow 2Mg^{2+} + 2OH^- + H_2$
- Corrosion product formation: $Mg^{2+} + 2OH^- \rightarrow Mg(OH)_2$
- Oxide formation: $Mg(OH)_2 \rightleftharpoons MgO + H_2O$
- Cathodic partial reaction: $2H^+ + 2e^- \rightarrow H_2$



- Z_f is the Faradaic impedance and C_d is the double-layer capacitance.
- R_f is the resistance of MgO (protective layer), which is much higher than Z_f .
- Film-free areas are small, and thus C_f is much higher than C_d , which C_f can be extracted from the high-frequency loop.



The impedance response is :

- A charge-transfer resistance loop at high frequency: Cathodic and anodic partial reactions.
- A diffusion impedance loop: Mg²⁺ through the porous Mg(OH)₂ layer.
- An inductive loop at low frequency: Adsorbed intermediate, Mg⁺, during Mg corrosion.

By using a Solartron 1250 frequency response analyzer in a frequency range of 65 kHz to a few mHz

Thank you for your attention.